Michael Funke (Ed.)

Factors in Business Investment

Papers of a Conference
Held at the Science Centre, Berlin,
Research Area "Labour Market and Employment",
September 1987

Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo

The Optimal Reaction of Production and Investment on Uncertainty

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1. AIM AND STRUCTURE OF THE PAPER

In this paper we show under which conditions uncertainty theory provides an unambiguous answer to the question whether firms in an uncertain environment will produce and invest more, the same, less than under certainty. In chapter 2 general models are developed which in principle apply to all decisions under uncertainty, though the decision variable is interpreted here mainly as production (for an overview see Aiginger 1987).

In chapter 3 we investigate parallel models about investment under uncertainty. In chapter 4 we use information out of the real world production process in industrialized countries to assess which models presented are more relevant.

The theoretical as well as the empirical results induce us to propose a new dichotomization of uncertainty situations into those of "petty" respectively "severe" uncertainty (chapter 5).

Though the results in this paper are arrived at by formal methods usually favoured by neo classical economists (expected utility maximization, density function as representation of uncertainty, micro level), the paper arrives at conclusions which can be interpreted as Keynesian in spirit (importance of flexibility, considerable impact of uncertainty on production, investment etc.).

2. A SURVEY ON MODELS OF DECISIONS UNDER UNCERTAINTY

In this chapter we will give a short overview on the models of decision of firms under uncertainty. Four general propositions are derived under which sufficient conditions are available to determine whether firms will produce more, the same or less than under certainty.

We will use Von Neumann-Morgenstern's Expected Utility Maximization. We will concentrate on passive models (where people can choose within a given framework without adapting it, information is given) and compare output decisions under uncertainty with output under certainty. The utility U depends on the variable Z (which could be understood as profits). Z itself depends on two variables X and Y (which usually are price and output). X is known under certainty (as X_0), in case of uncertainty a probability function about this variable - f(X) - is known. Y^+ is the optimal value of the decision variable resulting from the maximization in equation (1), $\bar{Y}$ is the optimal value of the decision variable in the corresponding uncertainty model (2).

$$\text{Max } U\,[Z(X_0, Y)] \rightarrow Y^+ \quad \text{(certainty maximum)} \qquad (1)$$

$$\text{Max } E\,U[Z(X, Y)] \rightarrow \bar{Y} \quad \text{(uncertainty maximum)} \qquad (2)$$

Proposition 1:
Linear technology ($Z_{XX} = 0$) plus $dY^+/dX > 0$ yields the following sufficient condition

$$U_{ZZ} \lesseqgtr 0 \rightarrow \bar{Y} \lesseqgtr Y^+ \qquad (3)$$

Proposition 1 tells us that risk aversion may be a sufficient reason for a negative influence of uncertainty on the decision variable. This effect is often cited in literature, however the simple relation "risk aversion/neutrality/loving implies lower/equal/higher output" is correct only under two very restrictive assumptions. The first is that under certainty the optimum value of the decision variable, Y^+, depends positively on the value of X.

The second assumption is that profits are linear in the decision variable. This is the case in the competition model under price uncertainty, but not under a monopolistic model for an output setter with non-linear costs. In this case risk aversion may not suffice to guarantee a smaller output under uncertainty, risk neutrality does not guarantee that uncertainty does not change optimal decisions.

Proposition 2:
A linear utility function ($U_{ZZ} = 0$) and technological concavity, neutrality, convexity ($Z_{YXX} < 0$, $Z_{YXX} = 0$, $Z_{YXX} > 0$) yield the following sufficient condition

$$Z_{YXX} \lesseqgtr 0 \rightarrow \bar{Y} \lesseqgtr Y^+ \qquad (4)$$

This proposition leaves aside risk aversion or loving, the effect of uncertainty now depends on technological conditions, like the cost and demand curve.

Up to now the models have assumed market clearing. Some variable adjusted ex post in a way to equal supply and demand. Equations (5 - 7) resp. (8 - 10) present a certainty model and a corresponding uncertainty model in which production y and demand x may differ, expected profits depend on the smaller of demand (x) or production (y) in equation (8).

Certainty model:

$$\pi = r(y) - c(y) \tag{5}$$
$$\pi_y = r'(y) - c'(y) \tag{6}$$
$$\pi_{yy} = r''(y) - c''(y) < 0 \tag{7}$$

Uncertainty model:

$$E\pi = \min\ [r(x), r(y)] - c(y) \tag{8}$$

$$\frac{\partial E\pi}{\partial y} = \underset{\substack{\text{marginal}\\ \text{revenue}\\ \text{under}\\ \text{certainty}}}{r'(y)} - \underset{\substack{\text{marginal}\\ \text{costs of}\\ \text{uncertainty}}}{F(y)\ .\ r'(y)} - \underset{\substack{\text{marginal}\\ \text{costs}\\ \text{under}\\ \text{certainty}}}{c'(y)} = 0 \tag{9}$$

$$\frac{\partial^2 E\pi}{\partial y^2} = r''(y)[1 - F(y)] - r'(y) - c''(y) < 0 \tag{10}$$

Proposition 3:

Given a certainty model of type 5 and an uncertainty disequilibrium model of type 8, uncertainty adds an additional marginal cost component which is positive (since $F(q)$ as well $r'(y)$ are positive). This yields for this type of model the unambiguous result of equation 11 (recall that $r''(y)$ is smaller than $c''(y)$ in the neighborhood of Y^+).

$$\bar{Y} < Y^+ \tag{11}$$

This proposition yields support for the above mentioned presumption of macroeconomists, that uncertainty will reduce output. Its most special case is where marginal revenue is constant: then output is maximized under demand uncertainty and a fixed price, a situation which could be labelled as "competition under demand uncertainty", as "uncertainty model with fixed prices" or as "stochastic rationing model" (Hymans, 1966; Malinvaud, 1980; Costrell, 1983; Benassy, 1983 and all the newsboy models in inventory literature). The unanimous result that production will be reduced stems from the expected costs of uncertainty: either production proves ex post to be higher than demand (implying high production cost the inventory cost may hopefully be reduced by further revenues from stocks) or production proves ex post to be lower than demand (implying foregone earnings and goodwill loss which may hopefully be reduced by the feasibility to backlog some part of unsatisfied demand). In any case expected costs are higher than under certainty. These extra costs of uncertainty - which are elaborated in Aiginger (1985) - are somewhat related to the arguments of "less-efficiency" and "noise signals" presented in the macro literature. Microeconomists however do not like models of this kind since model 8 assumes price stickiness in some ad hoc fashion and because the model is a partial model focusing only on the view of the producer under a given price (identical in the certainty and the uncertainty model).

A fourth channel for changing optimal production is given if it is possible to make a preliminary decision about the decision variable and then, after the veil of uncertainty is lifted, to revise this decision at some cost.

Proposition 4:

Suppose it is possible to make a preliminary decision $\bar{Y}$ and revise this upward (downward) at cost c_1 (c_2) then

$$c_1 \gtrless c_2 \quad \text{tends to imply} \quad \bar{Y} \gtrless Y^+ \tag{12}$$

3. INVESTMENT MODELS

The number of feasible investment models is larger than of those for the output or price dicisions. What is added are the possibilities that the input of either one or both (all) production factor(s) must be determined before the veil of uncertainty is lifted, that this choice can be binding for output or not (fix or variable factor output relation), the production factors may be substitutable ex ante only, or ex ante and ex post (putty-clay assumption, putty-putty assumption), to which is added the impact of further attributes of the production functions (e.g. returns to scale). There may also be uncertainty - besides that on price and demand - in respect of the production factors' pay (wages) or their performance (and efficiency). Since long term considerations are especially important for investment decisions, the pressure to introduce dynamic models is greater than for output decisions.

Out of the theoretically feasible variety of models we present only a few, for a more complete overview and for the exact derivation of the results see Aiginger 1987 (chapter 7).

If both inputs are to be decided ex ante and prices are fixed, then the input decision is arrived at by the same laws as under certainty - though in respect to the output chosen under uncertainty. That means if the output under uncertainty equals the output under certainty, then the inputs are equal, too. If output is smaller (maybe due to proposition 1 or 2 or 3) then - given a well behaved production function - both inputs will decrease.

If capital has to be decided ex ante in a model of demand uncertainty (and disequilibria due to a fixed price), and labour can be chosen ex post (saving wages in case of low demand), then a linear version of proposition 2 applies. Whether more or less than expected demand is produced, depends on the relation between capital costs (which are lost in case of low demand) and unit profit (surplus of price over wages per unit).

If uncertainty about prices exist, but firms can sell at the market price whatever produced, we have an equilibrium model fitting to proposition 2. Let us assume that capital has to be chosen ex ante, labour ex post and there are no limits to the substitutability of capital, than we derive at the result, that investment depends on the relation between the elasticity of substitution and the degree of economies of scale. If substitutability is relatively low and/or if the production function approaches the linear homogeneous production function, capital input will be higher than under certainty:

$$\frac{\partial(F_{KL}/F_{LL})}{\partial L} \gtrless 0 \text{ if } \sigma \gtrless (1 - \mu)^{-1} \qquad (13)$$

The economic interpretation of this outcome will be found in the fact that, when the factors can easily be substituted, a low capital input can be cheaply compensated for by means of ex post variable labour input; alternatively, given large diseconomies of scale, a contingently high price will lead to only a moderate increase of optimally profitable output. Generally it can be assumed according to the present model that capital input may be rather higher than under certainty. For example, given a substitution elasticity of 2, μ would have to be smaller than half in order to lead to a lower capital input. Not below a substitution elasticity of 4 might μ approach 0.75. For such seemingly realistic values of the scale parameter as 0.8 or 0.9, unrealistic substitution elasticities would have to be attained. If on the other hand the production function approaches linear homogeneous, then the possibility of reducing the capital input evaporates completely in this model of price uncertainty.

A minor change in the assumption can change the results. Kon assumes that the capital labor ratio has to be chosen ex ante. Ex post an optimal second step decision can be done, for less than full utilization of capital labour cost can be saved. The result now depend on a flexibility effect parallell to the last model, but a utilization risk effect is added which usually biases the investment downward. We have some sort of combination of proposition 2 and 3 and the results are ambigous.

Switching to dynamic models in principle does not change the story. If we model dynamic demand functions without disequilibria and without irreversibility we arrive at models of proposition 2 with - in general - ambiguous results. In Nickell's model for example optimal investment depends on the demand function with some more chances to bias capacity downward (though we know that other models would have given other results). If, however, we assume that investment can be revised upwards in case of strong demand, but downward changes of capital stock are limited to depreciation, we get a strong bias to a cautious dynamic investment strategy. The economic rational follows the spirit of proposition 3, namely that capacities may prove idle.

4. EMPIRICAL BEHAVIOUR

One of the critical facts theory points out and which has to be evaluated empirically is the degree of price stickiness in modern industrial production. Uncertainty will yield different decisions as compared with certainty if prices are sticky, albeit this has to be only a short-term stickiness. Only if prices are flexible in the sense of immediately offsetting demand shocks no disequilibrium costs have to be incorporated. We have demonstrated empirically by econometric and by survey methods that:

- industrial prices tend to reflect cost conditions and may be some price signals stemming from international markets, but do not react rapidly to demand shocks;
- prices seem to be less variable than quantities, especially the short-run fluctuations;
- price expectations are not less inaccurate as compared with production expectations (as they should be if the prices were an ex post control);
- price rigidities are at least as dominant in that sector of manufacturing for which a large number of enterprises, their small average size, etc., would suggest more competitive behaviour than for the rest; and
- asked about their response to a demand shock only 20 per cent of the firms in a survey cited price change as primary response.

On the other hand, quantities seem to be more flexible than assumed in most models, be it that production is an ex post control or be it that a preliminary production can be decided upon. Then, after demand is revealed, the decision variable can be partially adjusted (ex post flexibility); as indicators for a partial adjustment of quantities with a remaining part of disequilibria we found:

- the capacity utilization of industry as well as the inventory sales ratio fluctuates to a considerable extent, and surveys tell as that firms do consider these fluctuations as involuntary;
- output volume follows demand shocks closer than prices; and
- asked about reaction to demand shocks, 56 per cent of the firms labelled quantity changes a primary response, and 48 per cent reported changes in inventories (part of the disequilibrium is maintained).

In general, ex post flexibility seems to be greater than suggested in standard models. The distinction between ex ante variables, which have to be decided before the veil of uncertainty is lifted and ex post variables, which have to be decided (or which adjust) thereafter is not watertight.

However, ex post flexibility of quantities seems easier than that of prices (in contradiction to theoretical assumptions).

Ex post flexibility seems easier upwards than downwards. This seems especially true for input decisions, where this tendency for reported investment anticipations can easily be demonstrated on the micro and macro level. This asymmetry overcomes the theoretical tendency of uncertainty to increase optimal investment. From the theoretical viewpoint this source of bias is not very attractive. To model an asymmetry sounds ad hoc, if it is empirically true, the theoretical implications are trivial.

The mainstream model of competition with price uncertainty is exposed as an outsider in a real economy. Only 7 per cent of the firms consider it as relevant to their situation; a monopoly model with price as ex post control is chosen by 16 per cent. The remaining majority reported that prices are not an ex post control; quantity-setting and a partial ex post adjustment of the quantity set are considered as the most realistic models.

If we should draw a picture of a standard "representative firm" in modern industry we would do so in the following way: industrial firms have to decide on a preliminary production, they set up a cost price (or accept a market price). In case of demand shocks, output is partly adjusted, partly backlogs and/or inventories are changed, prices change slowly and in response to large shocks. Adjusting quantities upwards is easier/less costly than adjusting them downwards. At least sources three and four (marginal cost of uncertainty and asymmetric ex post flexibility) usually tend to bias (preliminary) decision under uncertainty downwards, technological concavity may add some other source of asymmetry.

Risk attitude is the most popular channel in the literature but most difficult to assess. If the results of entrepreneurs in a survey may be considered reliable, it looks as if entrepreneurs behaved as if they were risk-neutral for small, repeated decisions and risk-averse for large, one-shot decisions.

Using these findings (for detail see Aiginger 1987, chapter 6, 7, 8) for the production and investment model it looks as if models following the proposition 3 are of greatest relevance. For investment models in special we think that the technical possibilities to adjust the preliminary investment plan upwards and the necessary incorporation of the possibility that your capital stock may be underutilized will outweight the tendency shown in models of operationalization 2 that elements of substitutability will advise to provide a relatively larger capital stock under uncertainty to be able to meet eventually high demand.

5. "PETTY" vs "SEVERE" UNCERTAINTY

In general the result of the models invites us to find a new dichotomization of uncertainty situations. The dominant dichotomization of uncertainty literature into "risk" and "uncertainty proper" (according to the criterion whether probability functions about the uncertainty variable can be formed or not) is not very fruitful, since in the latter case only very crude rules of behaviour can be derived in a coherent and consistent way. The Keynesian view, that economic decisions are done in an environment much more complex than in an optimization problem where one certain variable is substituted by one for which a probability function is known, is nevertheless a useful warning. That no probability function can be assessed (or used implicitly) is an extreme alternative however, and precludes the economic analysis of a large area of economic problems.

We believe that it is important how the decision model is constructed, whether the importance of uncertainty will be considerable or minor, not whether we assume that probabilities can be assessed. If we construct models in which disequilibria exist and are not instantaneously closed by some ex post control, if we model the decision process as choosing between alternative techniques and degrees of flexibility, then we can use Neumann-Morgenstern's expected utility theory in general and probability functions and nevertheless describe a situation in which people behave "qualitatively differently" under certainty and uncertainty.

We tentatively propose that the real divide between uncertainty that matters and uncertainty with less consequences is whether there are chances to correct a decision (or at least to make errors in some way unimportant). This correction can either be a two-stage optimization process (short-run optimization for a given long-term optimization, e. g. for labour and capital), or it can be that the market price adjusts automatically yielding equilibrium for any quantity decision or that goods are durable so that unsold production can be used in the next period. We propose to label situations in which such adjustments are feasible as "petty" uncertainty, since the importance of uncertainty is mitigated to a large extent by these strategies. Models in which there are less strategies for ex post adjustments are labelled as "severe" uncertainty, since they usually result in disequilibria with important medium- or long-run consequences.

6. CONCLUSIONS

(1) We think that modern uncertainty theory can provide useful results about the effect of uncertainty on production and investment decisions. We derived four general propositions which can tell us about the impact of uncertainty depending on risk attitude, technological concavity, disequilibria and asymmetries in ex post adjustments.

(2) The effect of uncertainty depends on several characteristica of the models. If we assume equilibria in the sense that there is some variable (usually the price) which closes the gap between plans and the value of the uncertainty variable as revealed ex post, then the effect of uncertainty depends on circumstances not easy to evaluate empirically (third cross derivatives). If on

the other hand there are imbalances (production less or larger than demand, idle capacities) then the downward bias of production or investment under uncertainty is easy to establish.

(3) We follow during this paper the method of expected utility maximizing (EUM). This could be partly critiziced against the increasing amount of literature proving that individual behaviour does not follow EUM. We know about and accept this evidence, but we believe that most of our results could be even easier demonstrated if people do not follow this stringent logical system of EUM. Keynesians have always maintained that uncertainty just changes the rules, and that under "true" uncertainty (in contrast to risk) we should never use the assumption that people know probabilities. We claim that we can arrive at conclusions very near to Keynesian ideas by using the formal method of EUM. We can demonstrate the importance of flexibility, the fact that uncertainty significantly changes optimal decisions (even under risk neutrality). If we switch from the micro to the macro level it follows that growth and employment will depend under realistic conditions on uncertainty. It is the way in which we model the economy (equilibrium vs. disequilibrium, price or quantity adjustment, ex post parameters and their symmetry) which decides on the results and not the use of the formal method. If Keynesians accept this, they will be able to keep track in a world of mathematical economists (it remains a good stimulus if Post-Keynesians still maintain that uncertainty changes the model in an even more drastic way).

(4) To demonstrate the importance of the model chosen, we propose a dichotomization into "petty" uncertainty and "severe" uncertainty. Under "petty" uncertainty there is always a way to correct the decision after the veil of uncertainty is lifted and the economic consequences are not as large as under severe uncertainty. Severe uncertainty is characterized by important one shot decisions whith less possibilities to correct them afterwards. Economic consequences are large and there is a pressure to change the rules (the market, the behaviour, the production technique, the social system etc.).

Table 1

	Petty Uncertainty	Severe Uncertainty
	versus	
definitions	uncertainty is an intermediate problem, some variables have to be decided before the realization of X is known, some thereafter in a short run optimization or they adjust automatically	lack of ex post adjustments - no ex post control - price stickiness
characteristics	repeated (or small) decisions lack of serial correlation for realizations of X	one shot (large) decisions X serially correlated
	insurances	letal events (bankruptcy, dismissal)
	future markets	
	ex post flexibility, continuous adjustments	irreversibilities of investment and technologies
consequences	minor differences to certainty depending on facts difficult to evaluate (Z_{YXX})	important consequences (usually biasing down the optimal value of the decision variable); pressure to change the model to include new cost components and strategies
empirically testable conclusions for the relevance of the model	equilibria flexible prices and quantities uncertainty does not depress economic activity	disequilibria price stickiness uncertainty depresses economic activity

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Symbols used

π	profit
U	utility
Z	argument of utility function (e. g. profit)
X	random variable; EX = X_o (value of variable(s) under uncertainty)
Y	action variable
$\bar{Y}$, Y*	optimal decision under uncertainty or certainty, respectively
U_{YXX}, Z_{YXX}	suffixes signify partial derivatives
p	price ($\tilde{p}$ = p + g: extended opportunity costs = price + goodwill costs); p ≥ c; p ≥ c' (q)
c(q)	production costs (c' = marginal costs, c" = 2nd derivation with respect to q; c = constant unit costs)
x, q, s	demanded, produced, sold quantity
E	expectation operator
f(·)	density function (symmetrical, smoothly differentiable);
F(·)	distribution function of f(·), values between 0 and 1
r(x), r(q)	revenue function in dependence on demanded, on produced quantity
σ, μ	elasticity of substitution, scale paramter