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A NEW DICHOTOMIZATION FOR UNCERTAINTY MODELS

1. INTRODUCTION

... those who wish to talk about 'leaps into the unknown' ... ought to be investing a lot of effort in finding a reasonable coherent way to deal with true uncertainty. Otherwise a lot of territory has to be abandoned", Solow (1984).

The aim of the paper is to try to develope a new dichotomization of risky situations. On the one hand firms may work in an environment where there are no (or few) chances to correct a decision after it is done, this will be called "severe uncertainty", on the other hand there are situations where there are chances to correct it or adjust a first decision in various ways, this will be called "petty" uncertainty. This dichotomization could prove more useful than that by Knight, who on the one hand distinguishes between situations in which people can form a probability function about a random variable (risk) and on the other those where they cannot (uncertainty proper). In macroeconomic theory Keynesians and Mathematical economists are divided over the "nature" of uncertainty to a considerable extent.

In contrast to the risk-uncertainty divide our dichotomization between "petty" and "severe" uncertainty assumes that agents in either situation are able to construct a - however crude or subjective - probability distribution. The main distinction lies in the degree of flexibility after the veil of uncertainty has lifted. We think this to be a more fruitful distinction between qualitatively different situation than to declare that people under "true uncertainty" just behave differently ("leaps into the unknown"). Mathematical concepts can be used now in either situation and we can show that some results "Keynesian in spirit" can be arrived at even under Expected Utility Maximization (they can be attained also when some of its alternatives like Regret Theory, Machina's Theory, satisficing behaviour,

B. R. Munier (ed.), Risk, Decision and Rationality, 575–593.

Allais' or Hagen's approach are used).

The paper is structured as follows. In chapter 2 we discuss Knight's dichotomization, in chapter 3 we introduce our own. In chapter 4 we use the concept of Expected Utility Maximization allowing for different market structures and different decision variables as proposed in literature.

In chapter 5 we confront models with alternative strategic responses of firms after the veil of uncertainty is lifted to demonstrate the difference between severe and petty uncertainty.

In the final chapter we repeat that this is a preliminary version of the paper - in some respects a very preliminary. On the other hand some of the findings (for example chapter 4) had been part of earlier studies of the authors (see Aiginger 1985A, 1985B). The proposed dichotomization into "petty" and "severe" uncertainty is an idea in the first phase of its "product cycle" and very open for critics and discussion.

2. KNIGHT'S DICHOTOMIZATION AND WHY WE NEED AN ALTERNATIVE

The most popular dichotomization of uncertainty situations is due to Knight (1921). He claimed the term "uncertainty" (thereafter "uncertainty proper") for situations where no probabilities about a random variable can be formed and used the term "risk" to describe situations in which people can form a probability distribution for the uncertain variable. Over decades economists debated, whether this is a fruitful divide and which situation is a more adequate description for real economies. Keynesians and Post-Keynesians very much stress that uncertainty in economic life is of the "uncertainty proper" type, since "the future is really unknown", "people just do not know". Not even all feasible alternatives are defined and it is therefore very misleading to assume that people can form probability distributions in uncertain situations. Instead they behave "qualitatively different" under uncertainty, for example preferring liquidity, flexible techniques etc. Despite the plausibility of the Keynesian arguments modern decision theory (most prominently the Von Neumann-Morgenstern-Theory) nearly without exceptions refers to the "risk" concept. The reason for this is that the assumption of probabilities is the main condition for making the models operational. Exact comparisons and calculations between certainty and uncertainty and numerical calculations of results are possible only if an however crude assessment of the probability of situations is feasible.

The agnostic view that under "uncertainty proper" we just do not know, is one of the causes why Keynesian economists lost a lot of ground in the economic science especially at the time in which microeconomic foundations became important and optimization the all important precondition for research to be considered as academic. Let me cite Robert Solow, a leading Post-Keynesians as one who shares my view that speaking about "leaps into the unknown" is a fruitless strategy abandoning much ground for economists in general and Keynesians in special.

3. "PETTY" VERSUS "SEVERE UNCERTAINTY": ELEMENTS OF AN ALTERNATIVE DICHOTOMIZATION

We want to propose - see table 1 - an alternative dichotomization for situations (types) of uncertainty. On the one side there is a type of uncertainty where uncertainty is some sort of an "intermediate" problem. That means a decision about one part of the variables have to be done before the veil of uncertainty is lifted, some other variable(s) adjust thereafter. This type includes models

- where there is an ex post control, which adjust automatically (market price, output given by the demand curve);
- where there is an optimization process feasible for some variable after the realization of the random variable is known (short run profit maximization for the variable factor).

Related economic consequences (to that of ex post control) are given if a decision does not have an one shot character but is of a repeated nature, expecially if the realization of the random variable are not correlated over time or if there exist insurances and/or future markets and there are no irreversibilities. If some or all of these ex post strategies are feasible there will be no disequilibria between supply and demand at least not for some meaningful period. Since this type of uncertainty is relatively easy to cope with we will label it "petty uncertainty". The optimal decision parameters are different nevertheless from those under certainty, however probably not "too far". For example for models following the type of proposition 2, the third cross derivative of the objective function decides. We can conjecture that for firms with approximately linear costs the effect of uncertainty in such models will be a minor one.

On the other hand there is a type of uncertainty where there is a lack of ex post adjustments in some very broad sense. This lack of ex post adjustment starts with a lack of a formal ex post control in the model or with price stickiness, thereby generating disequilibria. The possibility of a final negative event like bankrupcy or dismissal is another one. Irreversibility of investment or the fixedness of a production technology chosen are further constraints. One single decision is crucially important so that later decisions in the next periods cannot change the fortune, sometimes even the risk cannot be insured. If some or all of these characteristics hold the economy will experience a lot of disequilibria between supply and demand and between factors employed and factors warranted. Firms will regard this type of uncertainty as especially unfavourable, since they do not have many chances to react to the realization of the random variable. We will therefore label it as "severe" uncertainty. The optimal decision will differ from certainty much more than for petty uncertainty, since a cost component is added in the uncertainty model (marginal cost of uncertainty, f. e. probability of excess demand or supply, information costs, bankrupcy feasibility) which does not even exist under certainty. Under "severe uncertainty" it seems very probable that optimal production is less than under certainty, according to arguments following proposition 3 (marginal costs of uncertainty) and 4 (less downward than upward flexibility). In general severe uncertainty generates a pressure to change the model to a larger extent then just to substitute a known value by a probability function. We would like to add information costs, goodwill and holding costs, probability of bankrupcy, cost of changing the technology etc.

Our distinction between "petty" and "severe uncertainty" resembles that between uncertainty and risk or between Expected Utility Maximization and Keynesian uncertainty. However we think it is necessary and feasible to use the formal structure of Von Neumann-Morgenstern to derive results for situations like risk or Keynesian uncertainty at least as a first approximation. If we have derived a preliminary qualitative result by this procedure we can still argue, that the situation may be "worse" than modelled insofar as people do not know the probability distribution

Table 1

	Petty Uncertainty	versus	Severe Uncertainty
definitions	uncertainty is an intermediate problem, some variables have to be decided before the realization of X is known, some thereafter in a short run optimization or they adjust automatically		lack of ex post adjustments - no ex post control - price stickiness
characteristics	repeated (or small) decisions lack of serial correlation for realizations of X		one shot (large) decisions X serially correlated
	insurances		letal events (bankruptcy, dismissal)
	future markets		
	ex post flexibility, continuous adjustments		irreversibilities of investment and technologies
consequences	minor differences to certainty depending on facts difficult to evaluate (Z_{YXX})		important consequences (usually biasing down the optimal value of the decision variable) pressure to change the model to include new cost components and strategies
empirically testable conclusions for the relevance of the model	equilibria flexible prices and quantities uncertainty does not depress economic activity		disequilibria price stickiness uncertainty depresses economic activity pressure to change "the rules"

exactly or at least not with great confidence (Falkinger, 1986), that there are extra costs of uncertainty not yet considered or that even the type of model used should be changed. All these factors, already stressed by Keynesians, will gain more acceptance if we have proved that under "severe uncertainty" behavior really changes even within the procrustus bed of models inherited from the world of certainty and treated by expected utility maximization.

4. A SURVEY ON MODELS OF DECISIONS UNDER UNCERTAINTY

In this chapter we will give a short overview on the models of decision of firms under uncertainty. Four general propositions are derived under which sufficient conditions are available to determine whether firms will produce more the same or less than under certainty. The propositions are published in Aiginger (1985A, 1985B) and are repeated here as a background for the following models either for petty or severe uncertainty.

We will use Von Neumann-Morgenstern's Expected Utility Maximization. We will concentrate on passive models (where people can choose within a given framework without adapting it, information is given) and compare output decisions under uncertainty with output under certainty. The utility U depends on the variable Z (which could be understood as profits). Z itself depends on two variables X and Y (which usually are price and output). X is known under uncertainty (as X_o), in case of uncertainty a probability function about this variable - f(X) - is known. Y^+ is the optimal value of the decision variable resulting from the maximization in equation (1), $\overline{Y}$ is the optimal value of the decision variable in the corresponding uncertainty model (2)

$$\text{Max } U\,[Z(X_o,\, Y)] \rightarrow Y^+ \quad \text{(certainty maximum)} \qquad (1)$$

$$\text{Max } E\,U[Z(X,\, Y)] \rightarrow \overline{Y} \quad \text{(uncertainty maximum)} \qquad (2)$$

Proposition 1:

Linear technology ($Z_{XX} = 0$) plus $dY^+/dX > 0$ yields the following sufficient condition

$$U_{ZZ} \lesseqgtr 0 \rightarrow \overline{Y} \lesseqgtr Y^+ \qquad (3)$$

Proposition 1 tells us that risk aversion may be a sufficient reason for a negative influence of uncertainty on the decision variable. This effect is often cited in literature (see Arrow, 1978, Gahlen et al., 1983 etc.), however the simple relation "risk aversion/neutrality/loving implies lower/equal/higher output" is correct only under two very restrictive assumptions. The first is that under certainty the optimum value of the decision variable, Y^+, depends positively on the value of X.

The second assumption is that profits are linear in the decision variable. This is the case in the competition model under price uncertainty, but not under a monopolistic model for an output setter with non-linear costs. In this case risk aversion may not suffice to guarantee a smaller output under uncertainty, risk neutrality does not guarantee that uncertainty does not change optimal decisions.

Proposition 2:

A linear utility function ($U_{ZZ} = 0$) and technological concavity, neutrality, convexity ($Z_{YXX} < 0$, $Z_{YXX} = 0$, $Z_{YXX} > 0$) yield the following sufficient condition

$$Z_{YXX} \lesseqgtr 0 \rightarrow \bar{Y} \lesseqgtr Y^+ \tag{4}$$

This proposition leaves aside risk aversion or loving, the effect of uncertainty now depends on technological conditions, like the cost and demand curve.

Up to now the models have assumed market clearing. Some variable adjusted ex post in a way to equal supply and demand. Equations (5-7) resp. (8-10) present a certainty model and a corresponding uncertainty model in which production y and demand x may differ, expected profits depend on the smaller of demand (x) or production (y) in equation (8).

Certainty model:

$$\Pi = r(y) - c\,(y) \tag{5}$$

$$\Pi_y = r'(y) - c'(y) \tag{6}$$

$$\Pi_{yy} = r''(y) - c''(y) < 0 \tag{7}$$

Uncertainty model:

$$E\Pi = \min [r(x), r(y)] - c(y) \tag{8}$$

$$\frac{\partial E\Pi}{\partial y} = \underbrace{r'(y)}_{\substack{\text{marginal}\\ \text{revenue}\\ \text{under}\\ \text{certainty}}} - \underbrace{F(y) \cdot r'(y)}_{\substack{\text{marginal}\\ \text{costs of}\\ \text{uncertainty}}} - \underbrace{c'(y)}_{\substack{\text{marginal}\\ \text{costs}\\ \text{under}\\ \text{certainty}}} = 0 \tag{9}$$

$$\frac{\partial^2 E\Pi}{\partial y^2} = r''(y)[1 - F(y)] - r'(y) - c''(y) < 0 \tag{10}$$

Proposition 3:

Given a certainty model of type 5 and an uncertainty disequilibrium model of type 8, uncertainty adds an additional marginal cost component which is positive (since $F(q)$ as well $r'(y)$ are positive). This yields for this type of model the unambiguous result of equation 11 (recall that $r''(y)$ is smaller than $c''(y)$ in the neighborhood of Y^+).

$$\overline{Y} < Y^+ \tag{11}$$

This proposition yields support for the above mentioned presumption of macroeconomists, that uncertainty will reduce output. Its most special case is where marginal revenue is constant: then output is maximized under demand uncertainty and a fixed price, a situation which could be labelled as "competition under demand uncertainty", as "uncertainty model with fixed prices" or as "stochastic rationing model" (Hymans, 1966; Malinvaud, 1980; Costrell, 1983; Benassy, 1983 and all the newsboy models in inventory literature). The unanimous result that production will be reduced stems from the expected costs of uncertainty: either production proves ex post to be higher than demand (implying high production cost the inventory cost may hopefully be reduced by further revenues from stocks) or production proves ex post to be lower than demand (implying foregone earnings and goodwill loss which may hopefully be reduced by the feasibility to backlog some part of unsatisfied demand). In any case expected costs are higher than under certainty. These extra costs of uncertainty - which are elaborated in Aiginger (1985A) - are somewhat related to the arguments of "less-efficiency" and "noise signals" presented in the macro literature. Microeconomists however do not like models of this kind since model 8 assumes price stickiness in some ad hoc fashion and because the model is a partial model focusing only on the view of the producer under a given price (identical in the certainty and the uncertainty model).

A fourth channel for changing optimal production is given if it is possible to make a preliminary decision about the decision variable and then, after the veil of uncertainty is lifted, to revise this decision at some cost.

Proposition 4:

Suppose it is possible to make a preliminary decision $\overline{Y}$ and revise this upward (downward) at cost c_1 (c_2) then

$$c_1 \gtreqless c_2 \quad \text{tends to imply} \quad \overline{Y} \gtreqless Y^+ \tag{12}$$

5. PRESENTATION OF SOME MODELS TO DEMONSTRATE THE DIFFERENCE BETWEEN PETTY AND SEVERE UNCERTAINTY

5.1 Competition with price uncertainty vs. demand uncertainty

In the competition model with price uncertainty (CPU, which is the mainstream model), uncertainty about the price does not effect the optimal production decision ($\hat{q} = q^+$) as long as the decision maker is risk neutral (which is assumed by maximizing expected profits).

In the competition model with demand uncertainty (CDU, which is considered as outsider due to reasons discussed in Aiginger, 1985A,B) uncertainty unambiguously biases optimal quantity downwards (in case of convex production cost under certainty, which are necessary to arrive at a unique optimal solution under certainty). The reason for this bias is that expected marginal revenues are equated now to marginal cost plus an additional cost component stemming from the (however small) probability that part of the production will not be sold. The difference is not due to the "prices vs. quantities" issue, but due to the degree of flexibility. In the CPU model price changes in a way that all production is sold, while in the CDU model no variable closes an eventual gap between production and demand.

The formulas read (where 1 is the function to be maximized, 2 gives the condition for the optimum, and 3 confronts the outcome with the certainty optimum) as following

CPU	CDU
$Y : q \quad X : p$	$Y : q \quad X : x$
(1) $E\pi = \int_0^\infty [pq - c(q)]f(p)dp$	(1A) $E\pi = p \min(q,x) - c(q)$
(2) $Ep = c'(q)$	(2A) $p = c'(q) + p \cdot F(q)$
	marginal costs of uncertainty
(3) $\bar{q} = q^+$	(3A) $\bar{q} < q^+$

5.2 Monopoly equilibrium vs. disequilibrium model

In the monopoly model there are on the one hand strategies in which entrepreneurs have to decide either about price or quantity before the veil of uncertainty is lifted, on the other hand there is a model in which firms have to decide about price <u>and</u> quantity before the true demand is known. The first two cases will be labelled as equilibrium models, since there is no quantity produced without being sold and no demand unsatisfied, while in the last model we arrive usually at an disequilibrium with either unsold products or demand unsatisfied. The results depend on the type of uncertainty (for example additive or multiplicative and within the second category also on the exact functional form, see Aiginger, 1985A,B).

In equilibrium models we have to assess the influence of uncertainty for six different cases. We can assume additive uncertainty, multiplicative uncertainty of type A ($p = g(q) \cdot u$), multiplicative uncertainty of type B ($q = f(p) \cdot u$) and for each of these three models we have to differentiate between price setting and quantity setting behaviour.

For the additive model with q-mode (quantity setting), optimal quantity under certainty and certainty are equal, the same is true for the multiplicative model of type A. For multiplicative models of type B, the question whether marginal revenue (with respect to q) is concave, linear or convex in u decides whether less, the same or more is produced under uncertainty.

For additive model with p mode (price setting) marginal costs (with respect to price) decide. When they are concave, linear, convex in u, then the price is smaller, the same, higher than under certainty. A similar condition exists for multiplicative uncertainty of type B, for type A an additional influence of the marginal revenue term is added. For constant marginal costs only in the last mentioned case, uncertainty changes the optimal price.

In disequilibrium model prices and output are jointly determined (before demand becomes known). Even with linear costs (which are assumed for simplicity) the optimal output differs under very likely circumstances from that under certainty due to two components. The first is a "price effect": under multiplicative uncertainty the optimal price is very likely above that under certainty (as shown by Aiginger, 1985A), while under additive it is smaller. In either case a second component due to the potential effect of unsatisfied demand or overproduction (marginal costs of uncertainty) is added, which biasses the decision downward. For the details see Aiginger (1985A).

The main difference between the equilibrium and the disequilibrium models again lies in the source of the effect. In the equilibrium models a different decision between certainty and uncertainty stems from the third cross derivative of either the cost function or the revenue function. Economists often tend to assume this to be zero since they simple do not possess arguments why it should be either positive or negative. In case of disequilibrium models, the decision under uncertainty will differ more likely from that under certainty. Even with linear costs a higher or lower price will influence the output decision (first component), and the marginal cost of uncertainty stemming from the potential unsatisfied demand/or the potentially unsold production will divert it additionally from the certainty optimum. Most probable in the downward direction, it can be argued.

5.3 Optimal capacity in competitive markets - Hartman & Nickell models vs. Kon's model

The input of labour is decided by the maximization of short term profits (Π_{SR}), the long term profit (after choice of optimal labour in the short term function) is given in equation (2).

Equation (3) shows the result (following proposition 2). Since the second term decides whether more or less capital input is required under uncertainty, Hartman shows that this finally depends on the relation between the elasticity of substitution δ and the diseconomies of scale (the scale parameter μ is always assumed to be less than one). If the substitution elasticity is relatively large a rather small capital input may suffice (since adjusting labour will do the rest), but in general unrealistically high values of δ will be needed for plausible values of μ, so that in general the model tends to predict that capital input should be larger under uncertainty.

$$\Pi_{sr} = p \, . \, F(K,L) - wLF_K, F_L > 0, \text{ F is concave} \tag{1}$$

$$\Pi_{lr} = p \, . \, F(K,\tilde{L}) - w\tilde{L} - iK = g(K,p,w) - iK \tag{2}$$

$$Z_{YXX} = \Pi_{Kpp} = \underbrace{-F_L}_{-} \cdot \underbrace{\frac{\partial(F_{KL}/F_{LL})}{\partial L}}_{\text{see equ. (4)}} \cdot \underbrace{\frac{\partial\tilde{L}}{\partial p}}_{+} \tag{3}$$

$$\frac{\partial(F_{KL}/F_{LL})}{\partial L} \gtrless 0 \text{ if } \delta \gtrless (1-\mu)^{-1} \tag{4}$$

Kon's model allows free choice of the capital-labour-ratio ex ante, after prices are known the rate of utilization of capacity may be chosen (which means that labour costs can be saved if optimal production is lower). Equation (5) denotes the short term profit function (where $L_0 = \beta K_0$ denoted the maximum labour constraint), equation (6) is the long term profit function. The overall result now depends on two factors. The "flexibility effect" corresponds to that effect which is solely decisive in Hartman's model (tending to higher capital input), a "utilization risk effect" is added which under the assumption of CES functions will always decrease optimal capital input. Conditions for the overall results can be formulated (proposition 5). In general now it is more likely that a smaller capital input is chosen. The rationale for this is that while capital may be useful for eventual large production, this advantage has to be balanced against idle capacities (which are not used due to economic reasons after prices are known).

$$\Pi_{sr} = p.F(\theta K, \; \theta L_0) - w\theta \; L \Rightarrow \tilde{\theta} \tag{5}$$

$$\Pi_{lr} = p.F[\tilde{\theta}(p,K,L)K, \tilde{\theta}(p,K,L) | L] - w\tilde{\theta}(p,K,L)L - iK \tag{6}$$

"flexibility effect" (Kon, 1983)

$$F_{KL} \gtrless 0 \quad \text{tend to lead to } \bar{K} \gtrless K^* \tag{7}$$

Proposition 5: Kon (1983), p. 188

$\bar{K} \geqslant K^*$ if $h' + zh'' \leqslant 0$ and $F_{KL} \geqslant 0$

$\bar{K} \leqslant K^*$ if $h' + zh'' \geqslant 0$ and $F_{KL} \leqslant 0$

where $F(K,L) = h\,[f(K,L)] = h(z)$

f ... linear homogenous production function

h(z) has characteristics $h'(z) > 0$, $h''(z) < 0$

5.4 One period vs. infinite horizon models

In one period models unsatisfied demand is lost and overproduction has no value. In multiperiod models unsatisfied demand (if it can be backlogged) may be met in the next period, inventories can be carried over to next period. Uncertainty in the second case is therefore less "severe", this will be shown for a simple model with linear costs and fixed price. In the one period model (see equation (1), a linear analogon to the CDU model in chapter 5.1), less than expected demand is produced as long as the profit share (p-c) is not larger than production cost (c)

$$E\Pi \;= p \cdot \min(x,q) - c \cdot q \tag{1}$$

$$F(y) = \frac{p-c}{p} \Rightarrow \bar{q} < Ex \text{ if } p - c < c \tag{2}$$

In a multiperiod model, with holding respectively goodwill costs, h(.) and g(.), a part of inventories and part of unsatisfied demand (a, b) can be transferred to the next period. The technique of recursive equations allows us to derive a functional relation, where V(I) is the maximum expected profit due to an inventory I (where q = y-I). The maximization yields equation (4). It can be shown that in general the consequences of both high and low output are of less significance than in the static case, since all disequilibria can be used in the next period (the extent depends on the durability of the goods and on the feasibility of backlogging). The results shown for linear models hold also for non linear models.

Uncertainty, we can conclude, tends to change the optimal decision a lot, if it refers to a once for all decision, if uncertainty comes in a way which can be modelled as repeated drawings out of an identical distribution its effect is smaller. Unsold products can be used, unsatisfied demand can be met tomorrow.

$$V(I) = \max_{q\geq 0} \int_{0}^{q+I} \{[px-h(y-x)] + \alpha V[a(-x+y)]\} f(x)dx + \int_{q+I}^{\infty} \{py+bp(x-y)-g(x-y)+\alpha V[b(-x+y)]\} f(x)dx-cq \quad (3)$$

$$F(y) = \frac{(p-c)(1-\alpha b)+g}{p+g+h-\alpha b(p-c)-\alpha ac} \quad (4)$$

5.5 Dynamic investment decision with and without irreversibility

5.5.1 Without irreversibilities

The firm maximizes its expected discounted net earnings. The planning horizon consists of three periods: to begin with, the first "equilibrium period" when demand rises with a known trend (β^0). This is followed by a non-recurring change in the growth path about which a probability distribution only is known (β_1 - the rate of demand increase prevailing from t on - is the random variable). The effect of this change lasts until the date t + m (m being the delivery lag), thereafter the actual capital stock will have adjusted to the rate of growth, known since the advent of the change. From this date a new "equilibrium period" prevails.

The condition for maximization of expected earnings is the outcome of equation (1). On the left side we find the (expected) marginal net earnings before the change in the trend of demand (N_Y^0), during the delivery period (EN_Y^1), as well as thereafter ($E\Pi_Y$). On the right side we find an "extended" capital cost term (see Nickell, p. 98 f.), with the "customary" capital costs (with the elements interest rates, depreciation and inflation) assumed to be constant (c). The equalization of expected revenue and capital costs may be reformulated in equation (2).

General condition:

$$g(t)\ N_Y^0 + [1-g(t)]EN_Y^1(\beta_1) + \theta(t-m)E\Pi_Y = \theta(t-m)q+c \quad (1)$$

$$g(t)\ N_Y^0 + [1-g(t)]EN_Y^1(\beta_1) = c + \theta(t-m)(q - E\Pi_Y) \quad (2)$$

Without irreversibilities the second part of the right side of (2) is dropped, since the free variability of capital stock makes it possible to equate the value of an additional unit after completion of the uncertainty period with its costs ($\Pi_Y = q$)

without irreversibility, with uncertainty:

$$g(t)\ N_Y^0 + [1 - g(t)]\ EN_Y^1\ (\beta_1) = c \qquad (3)$$

without irreversibility, without uncertainty

$$g(t)\ N_Y^0 + [1 - g(t)]\ N_Y^1\ (E\beta_1) = c \qquad (4)$$

The only difference between certainty and uncertainty is that between the expected marginal revenue and the certain marginal revenue during the delivery lag.

The effect of uncertainty depends, also in the dynamic model, on the concavity (linearity, convexity) of the marginal revenue with respect to the random variable (proposition 2).

$$N_Y^1\ (\beta_1)\ \begin{Bmatrix} \text{concave} \\ \text{linear} \\ \text{convex} \end{Bmatrix}\ \text{in}\ \beta_1 \Rightarrow \bar{y} \lesseqgtr y^+ \qquad (5)$$

5.5.2 The case of downward irreversibility

The absence of a second hand market for investment goods (downward irreversibility of the investment decision) has its impact on maximization insofar, as the expected discounted yield of the last unit of an investment programme ($E\pi_y$) is now smaller than its cost (q) and the second part of the right side in equation (1) turns positive. The optimal investment path is then always lower under irreversibility than under full reversibility, whenever there is even the smallest likelihood that a capacity unit may not be used. The effect of irreversibility can enter (given concavity in the model with reversibility) as an additional factor for a lowering of the investment path under certainty, or counteract its increase (in case of convexity of the marginal yield in respect of β).

With irreversibility and uncertainty

$$g(t)\ N_Y^0 + [1-g(t)]EN_Y^1(\beta_1) = c + \underbrace{\theta(t-m)(q - E\Pi_Y)}_{>\,0} \qquad (1)$$

$$\bar{y}_{IRREV.} < \bar{y}_{REV.} \qquad (2)$$

IRREV., resp. REV. ... signifies impossibility, resp. possibility of negative gross investments

The question of the irreversibility of investments is extensively dealt with in literature, where widely different model specifications and methods of solution were chosen. As a rule irreversibility distorts investment decisions downward.

6. PRELIMINARY CONCLUSIONS AND EXISTING PROBLEMS WITH THE NEW DICHOTOMIZATION

(1) The dominant dichotomization of uncertainty literature into "risk" and "uncertainty proper" (according to the criterium whether probability functions about the uncertainty variable can be formed or not) is not very fruitful, since in the later case only very crude rules of behaviour can be derived in a coherent and consistent way. The Keynesian view that economic decisions are done in an environment much more complex than in an optimization problem where one certain variable is substituted by one for which a probability function is known, is nevertheless a useful warning. That no probability function can be assessed (or is used implicitly) is an extreme alternative however, and precludes economic analysis to a large area of economic problems.

(2) We believe that it is important how the decision model is constructed, whether the importance of uncertainty will be considerable or minor, not whether we assume that probabilities can be assessed. If we construct models in which disequilibria exist and are not instantaneously closed by some ex post control, if we model the decision process as choosing between alternative techniques and degrees of flexibility than we can use Von Neumann-Morgenstern's Expected Utility Theory in general and probability functions and nevertheless describe situations in which people behave "qualitatively different" under certainty and uncertainty.

(3) We propose that the real divide between "uncertainty that matters" and uncertainty with less consequences is whether there are chances to correct a decision (or at least to make errors in some way unimportant). This correction can either be a two stage optimization process (short run optimization for a given long term optimization, f. e. for labour resp. capital), or it can be that the

market price adjusts automatically yielding equilibrium for any quantity decision or that goods are durable so that unsold production can be used in the next period. We propose to label situations in which such adjustments are feasible as "petty" uncertainty, since the importance of uncertainty is mitigated to a large extent by these strategies. Models in which there are less strategies for ex post adjustments are labelled as "severe" uncertainty, since they usually result in disequilibria with important medium or long run consequences.

(4) We showed some examples for either kind of situations:

- In the competive model with price uncertainty (where ex post prices clear the market), decisions under uncertainty and certainty are identical (for risk neutral firms). In the competive model under demand uncertainty (with disequilibria) firms produce less under uncertainty.

- In the monopoly model with market clearing there is little room for different behaviour under certainty and uncertainty. The outcome depends on the "technological concavity", where the third cross derivative of revenue and cost functions (about which we do not know much empirically) decides. In the monopoly disequilibrium model under nearly all circumstances the decisions will be different between certainty and uncertainty (due to a component labelled marginal costs of uncertainty which represents the expected cost of unsold production or unsatisfied demand).

- In models in which disequilibria (stocks or backlogged demand) can be transferred into the next period, decisions are more similar to certainty than in those where they are "lost". This stems from the fact that part of the "marginal costs of uncertainty" can be recovered in the dynamic context (goods can be sold, demand satisfied).

- If a preliminary production decision can be partly revised in the light of new information, if investment goods can be sold in a second hand market, the outcomes are more similar than with inflexibility and irreversibility of investment decisions.

(5) In all these cases Expected Utility Maximization is used and probabilities are assumed to be assessable. It comes from the kind we structure the economic model, whether uncertainty matters to a minor or to a larger extent, not from the question whether probabilities can be assessed or not.

(6) We proposed a new tentative dichotomization. We want to stress its shortcomings.

- A dichotomization to be useful should be along one criterium, optimally defined in a general model which covers all the specific cases as submodels. This is not done yet and may not be feasible at all. The decisive criterion "degree of ex post adjustments" has various dimensions (market price adjustments, second stage decision processes, flexibility, time). A long way is to be gone before we will get a formal general criterion. I would like to invite fellow economists to join me on this way.

- Whether the impact of uncertainty is considerable or minor should finally not be evaluated according to the criterion whether the decisions are similar or different or whether it depends on facts we know about or not (like the third cross derivative of a cost function). The final criterion should be expected profits.

(7) Nevertheless, I think I succeeded to demonstrate that results Keynesian in spirit, namely an important and assessable influence of uncertainty (even for risk neutral firms) could be demonstrated by mathematical tools, by accepting that people can assess probability functions and modelling the environment in a more realistic way. Fundamental Post-Keynesians will maintain that uncertainty changes the model and behaviour in an even more drastic extent, and they are probably right.

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