

Production and Decision Theory under Uncertainty

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Part I

Introduction

1 The Impact of Uncertainty – A Theoretical Topic of Great Relevance for Real-world Behaviour

1.1 THE AIM OF THE BOOK: CONFRONTATION OF THEORIES WITH EMPIRICAL EVIDENCE

The book addresses two questions. The first is: Under what conditions can optimally formed reported expectations of firms (like sales or investment anticipations) deviate from realizations (like actual sales or investment) even in the long run (on average)? The phenomenon of – usually downward – biased reported expectations is well documented in empirical surveys on inflation, export sales or investment ‘expectations’, for consumers, firms and sometimes even for macroeconomic forecasting institutions.¹ The ‘Rational Expectations’ Hypothesis as developed by Muth (1961), dominating macroeconomic literature in the specialized version of ‘expectations as conditional expected values’ suggests that the equality of the means of expectations and realizations should be the very lowest benchmark for rationality. We try to find under what circumstances ‘economically rational’ expectations should deviate consistently from realized values or from expected values.

The second question is: Is there a systematic difference between optimal decision under certainty and uncertainty, and especially do these differences show some general structure, for example does uncertainty bias the optimal value in the upward or the downward direction? We focus mainly on the optimal production decision, with less intensity on the price and input decisions. Since it is hopeless to assume that optimal decision will change under uncertainty in one direction under all logical circumstances and for all variables, we will first try to structure the theoretical models in a way that shows us what features of the models matter, then we will try to find out what can be told empirically about the crucial variables for modern industrialized countries.

This attempt to structure theoretical models and assess their relevance empirically is probably a rather singular feature of the present book. Most

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theoretical books on uncertainty stop many steps short of empirical assessment, and some restrict empirical evidence to experiments (e.g. Kahneman & Tversky 1979). On the other hand, most empirical work (for example macro-economic models but also empirical work by business economists) stops far short of addressing questions of uncertain environment in any explicit way (the unexplained part may be considered to stem from uncertainty but it may also be due to incomplete specification, etc.). The attempt to bridge the gap between theory and actual behaviour may seem unsatisfactory from both points of view. Its justification lies probably in the increasing importance of uncertainty for our economies since the mid-1970s, following the first oil crisis, but also due to political development.

Aiming at a realistic assessment and arriving at some dichotomies between the compelling logic of neoclassic equilibrium theory (or that of prevalent theoretical thought in general) and empirical facts of presumably suboptimal behaviour, we shall in Part II either bypass the question of the course of action actually followed or we may in fact decide in favour of the second course. But this second course need not necessarily be irrational: as a rule it can be rationalized by making certain assumptions on market form, rigidities, cost of collecting information, etc. The danger of arguing *ad hoc* or being accused of *ex post* rationalizing must be faced.

Here are some examples of deliberately dwelling on such dichotomies:

- In the competition model it is solely the model of price uncertainty that is theoretically unassailable, yet the model of quantity uncertainty (where firms have to choose a production level before quantity demanded becomes known, see 5.2.4) is not *a priori* discarded. It can be interpreted as a competition model of short-term price rigidity, as a competition model under short-term fixed market share, as a description of oligopolistic behaviour, or of markets under price regulation, as a fixed price model under stochastic rationing or as a 'specific' reaction, particularly under considerable uncertainty. This model is presented in Part II and the objections are discussed (5.1), in Part III we shall seek information on the empirical relevance of price rigidity – essential for this model – in near competitive markets (11.5).
- In the competition model under price uncertainty, risk aversion decreases optimal output. This distortion disappears if there are future markets for all products (Holthausen 1979). The present study is a partial analysis, rendering the entrepreneurial decision under uncertainty from the entrepreneur's point of view without enquiring into its consequences for market equilibrium or investigating the conditions of attainment of the equilibrium with and without future markets.² A possible step from the *ex ante* decision towards a market equilibrium is the *ex post* flexibility of the production decision. This is discussed in chapter 8 (Part II). In Part III, substantiations are sought for flexibility in

output decision on the one hand, and on the other hand some indications on whether it is market equilibria or disequilibria that are the rule in the industrial commodities markets (chapters 10–12).

- Three modes of behaviour are possible in monopoly under uncertainty: price setting, quantity setting, and price and quantity setting before the uncertainty about the demand curve clears. These behaviour patterns could (under certain assumptions) be ranked according to expected utility, and description might be confined to the one promising maximum expected utility. We present all these three behaviour patterns side by side in Part II, letting the empirical material decide on the relevance of the models.
- We consider the situation of a seller on one side of the market, i.e. a so-called partial analysis. It is difficult in an uncertainty model to interrelate the two sides of the market because there is the need endogenously to explain – beside the price – also the stochastic processes at work in the market. Failure to perform the consolidation restricts the persuasiveness of the findings. To be compelling, they need additional untested assumptions to be made – a highly unsatisfactory situation for the theorist.

While deliberately setting limits to the level of theoretical aspirations we also recognize limits to our empirical aims. The empirical findings are derived from market performance, pointing to conceptual differences from individual optimization in the theoretical part. Neglecting reference to the distortive power of possible or actual problems of aggregation the empirical sections aim only at evaluating the various theoretical models' plausibility or at obtaining indications about critical parameters without hope of explaining isolated, empirically interesting problems. Thus, in explaining prices' determination or inventory fluctuations we forgo the introduction of *ad hoc* justifiable auxiliary variables for the maximization of the coefficients of determination. We try to remain within the framework of the explanatory variables used in the theoretical model and to establish if they play their assigned role in practice.

The theoretical models are limited to business planning (although, as mentioned, even expert and consumer forecasts are distorted); the empirical results are mostly derived from data on the Austrian industry, supplemented where available from German, American and Japanese sources.

1.2 STRUCTURE

In the theoretical part (Part II), we discuss models dealing with the impact of uncertainty on the firm's production, price, inventory and input decision. In the first place the concept of reported business expectations as an optimal plan under uncertainty is developed and correlated with the

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common interpretation of expectation, in particular with the mainstream operationalization of the so-called Rational Expectations' Hypothesis (Muth 1961), as a conditional expected value.

The firms are assumed to be 'Neumann – Morgenstern Utility-Maximizers' (expected utility hypothesis), not because of its supposed closeness to reality but because of its operationality and its strict demands on the rationality of the economic units. That hypothesis will be a stringent standard for the rationality of 'distortions'.

Alternatives to the 'Neumann – Morgenstern utility theory' are dealt with in chapter 3. Among them are the traditional 'safety-first principles' (quite likely often applied as rules-of-thumb in daily practice) as well as the Prospect Theory, derived from the empirically ascertained infringements of the Neumann – Morgenstern axioms, as well as the Regret Theory or Machina's fundamental contribution. More than half a century ago Knight (1933)³ proposed to distinguish between 'risk' and 'uncertainty'. In risk he held it possible to estimate probabilities (the present study treats the risk case mainly by this distinction) – but not in uncertainty proper. Keynes and the Post-Keynesians are rather concerned with uncertainty (as dealt with at length by Rothschild, 1981) or – as we should say since the term of 'uncertainty' has largely displaced risk in the literature – with uncertainty proper, when largely new or unique constellations are at issue and when in consequence the economic actors can hardly have any knowledge of the probabilities. We propose to show how the concepts of risk and uncertainty proper despite their seemingly basic differences can in fact have mutually fertilizing effects. Uncertainty proper, for example, contributes potential rationality of price rigidity and emphasizes the advantages of flexibility. Among the possible model constructs in a Neumann – Morgenstern universe this book gives comparatively more room than in published literature to models featuring price rigidities or emergency strategies (*ex post* flexibility of quantity). We propose a new dichotomization of uncertainty models in section 9.2, where the applicability of a certain method (Neumann – Morgenstern's Utility-Maximization) is not the decisive criterion, but the structure of the model (what variables are fixed and what are endogenous). We label these alternatives as 'severe' versus 'petty' uncertainty.

Most results of the theoretical models may be interpreted as special cases of one of four general models (chapter 4). Best known in literature is the fact that risk aversion combined with linear technology (Z_{XX} equals zero, Z being the argument of the objective function, X the random variable, suffixes representing partial derivatives) will lead to downwards distortion of the optimal value of the action parameter under uncertainty, \hat{Y} , when compared with that in the certainty model, Y^* . We, on the contrary, shall concentrate on the case of risk neutrality – not as though risk aversion were *a priori* to be excluded, but rather because the preponderant attention paid in literature to the risk attitude evokes the impression that uncertainty

had no influence on decisions under risk neutrality. In such a case a 'concave technology' (in the sense of $Z_{YXX} < 0$) or marginal costs of uncertainty as will result from the chance – increasing with rising y – of unsaleability of output, lead to this distortion. These (second and third) classes of determinants for distortions are not much dealt with in the literature in the generality here presented (though special cases are discussed). The fourth emerges if preliminary decisions can be revised more easily (less costly) upwards than downwards.

Three branches of theories deal with the effect of uncertainty on the entrepreneur's decision (see chapters 5–7); they are distinguished by the action variables, but also by the ruling assumptions concerning random variables and the general setting (e.g. concerning price flexibility).

The theory of firms under uncertainty (chapter 5) deals primarily with one-period decisions concerning optimal price and output but excludes the effect of stocks on that decision. Attention is concentrated on the influence of market structure, cost curves and the specific modelling of the types of uncertainty. Other than mainstream literature, we do not *a priori* exclude the competition model under quantity uncertainty, because among other things, its effects, and in particular the non-clearance of the markets due to price rigidity, could also be typical for a wide range of oligopolistic markets, monopolistic competition or regulated markets. Among the monopoly markets (5.2) we also deal with a type of uncertainty that is neglected in the literature, because of the dominant position held by Leland's (1972) study.

The inventory theory (chapter 6) aims particularly at considering revenues and costs of future periods, thus representing a necessary complement to the theory of firms under uncertainty. We simplify by mostly implying linear costs and fixed prices where alternative market forms or types of uncertainty are left aside. The present study concentrates on determining the optimal stock on hand (stocks after output, before demand), it accepts constant unit revenues and costs as well as identical, independent distributions over demand (i.i.d.).⁴

In contrast to most writings on the subject, we do not focus on availability and the uniqueness of the solution (they are given due to the simplicity of our assumptions) but on the question whether the optimal stock on hand lies above or below expected demand.

We want to show which parameters act on this problem thereby modifying the one-period models. Contrary to the ruling implicit and explicit prejudice that inventory should rise with uncertainty, the opposite seems more plausible.

That prejudice has led to some writers imputing – without specifying the assumption – a special case (the lost-sales case) and in some cases even an error of omission is traceable through study after study, distorting the findings in the direction of the prejudice (see appendix 4).

The literature on optimal choice of production inputs (chapter 7) under uncertainty draws attention to production techniques. The outcome of

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the models depends on the specific production function and on the question: Which factors are to be chosen before and which after the veil of uncertainty has lifted?

A further step to increasing the models' realism lies in being able to modify the production decision after uncertainty has lifted at least in part (or at higher cost). The tendential effect of this *ex post* flexibility is demonstrated by means of two models (chapter 8).

Chapter 9 summarizes the findings of Part II and elaborates those critical parameters and assumptions on which the effect of uncertainty depends. These critical assumptions and parameters will be tested in empirical analyses. A dichotomization of uncertainty models into ones of 'petty uncertainty' and others of 'severe uncertainty' is proposed, in which mainly the availability of *ex post* controls – not the ability to construct a probability function – is the dividing criterion.

Chapter 10 is the bridge between the theoretical part of the book and the empirical. In particular, we characterize the role empirical evidence will play, which is not always that which purists would like to be able to achieve. The evidence does not come from carefully planned experiments but from surveys and time series gathered for other than theoretical purposes. No definite answer on the validity of the models can be expected, but hopefully we get some hints as to which models are more realistic than others.

Part III examines which of the models offered by theory are better able to describe the production process in industry. This is done in part by empirically checking the assumptions of the models and in part by reviewing whether their implications are empirically valid. Thus we examine the empirical determinants of net inventories, the problem of rigidity or flexibility of the action parameter determined in the initial decision.

The influence of durability on the production decision calls for thorough empirical examination because, first, the theoretic models are extremely diverse and secondly because the tendential result depends on the empirically established range of certain parameters. The empirical relevance of backlogging in particular, the relative costs of storing and bottlenecks as well as the relative insignificance of speculative stocks are important for the explanatory power of the models (chapter 11).

Some of the theoretic models assume short-term price rigidity, others stipulate market clearance by *ex post* variable prices. The real rigidity of prices in industry as well as the other implications of equilibrium models and disequilibrium models are scrutinized in chapter 12.

Chapter 13 tests whether the output decision, and the investment decision is equally (at even cost) variable upwards and down. The asymmetry of *ex post* flexibility as well as the possible irreversibility of the investment decision were assessed in the models to be important factors of a potential distortion of decision under uncertainty.

Part IV – which contains only chapter 14 – tries to summarize the important findings and restraints concerning the whole complex under investigation.

Appendix 1 gives a survey of abbreviations repeatedly used in the text; appendix 2 describes and discusses the problems and pitfalls of information gathered by surveys; appendix 3 demonstrates that the famous 'policy impotence result' does not hold with asymmetric losses due to expectational errors; appendix 4 gives an overview on the incorrect treatment of backlogged orders in management science literature; and appendix 5 presents the complete text of a survey frequently cited in the text.

1.3 THE INNOVATIVE CONTENT OF THE TREATISE

It is *not* the objective of this book to add to the countless models of entrepreneurial decision under uncertainty, a few more, distinguished by alternative assumptions concerning an action variable or uncertainty variable (demand, price, efficiency, input availability, etc.). What may be slightly innovative here is the somewhat more distinctly articulated modelling of productivity uncertainty (section 5.3), *ex post* flexibility (chapter 8) and a correction of current notions about revenues from backlogged orders (sections 6.5 and 6.6).

The envisaged innovation is an attempt to structure the numerous models by general types (propositions 1–4), to assign contradictory findings to implicit or explicit assumptions and to work out economic principles that had led to the models' specific outcomes.⁵ The regularities, once established, are empirically tested for closeness to reality.

A considerable shift of emphasis *vis-à-vis* mainstream writings will be observed in this volume. Within the general types of models the problem of risk aversion dominates extant literature⁶ (proposition 1 or operationalization A in chapter 4), where as the third cross derivative of the argument of objective function (Z) to the action and the uncertainty variable (Z_{YXX}) has as a rule been neglected: proposition 2 (operationalization B) has never, to my knowledge, been developed as a general possibility for distortion in models of firms theory under uncertainty, although – mathematically – it trivially follows from the well-known Rothschild – Stiglitz condition. The mathematically trivial difference between U_{YXX} and Z_{YXX} has an economic consequence, *viz.* the shift of the argument from (subjective) risk behaviour to technological (objective) decision determinants. This shift is meant to prevent the effect of uncertainty from automatically being reduced to risk behaviour about which, incidentally, not much is known (cf. Lippman & McCall 1981). Under risk neutrality, too, uncertainty theory is a large and interesting subject, and there cost and market conditions determine the outcome, thereby making it accessible to empirical enquiry. Of course, the equilibrium theorist has to love risk aversion since it will often assure the existence and the uniqueness of the results; however, in the present partial analysis we resist the temptation.

The second shift of emphasis lies in our moving from equilibrium models (meaning models with *ex post* control bridging the imbalance between

planning and realization of the uncertainty variables) to disequilibrium models where, for example, planned output and demand are not equated automatically by some *ex post* control.

The latter are known in literature, but they are marginally dealt with within standard theory.⁷ In this study they receive extensive consideration and are the subject of a rather general condition (preposition 3). Uncertainty creates an extra cost component (as compared with certainty) which is labelled as 'marginal costs of uncertainty'. Now marginal revenue is equated to marginal costs plus this extra component, thus leading to 'lower' optimal values under uncertainty.

This condition, too, is obviously implicit in many models, but it is neglected in literature as a general source for biasing the optimal decision downward.

Finally, this study seeks some reconciliation between diverse approaches in economic thought.

Firstly, we attempt to lessen the gap between mathematically oriented standard theory, which treats of uncertainty in the framework of Neumann – Morgenstern Utility Theory (thus basing itself on Knight's 'risk situation'), and the Keynesian view of 'true' uncertainty (wherein the economic actor is unable to evaluate the probabilities, and acts 'quite differently'). We will therefore incorporate some of the 'stylized facts' emphasized in the Keynesian view, such as price rigidity or the flexibility issue into the Neumann – Morgenstern Utility-Maximizer's model construction.

Secondly, we aim to reconcile the Rational Expectations' Hypothesis (Muth 1961) with the Keynesian system and also with the empirical findings from business surveys. This is done by underlining the original more general version of Rational Expectations' Theory, which did not rely on linear objective functions. If the objective function is not linear then a continuing divergence between (point) expectations and realizations is not contradicting the hypothesis. If, however, the objective function is linear, which is the operationalization dominating the application of the hypothesis to economic policy, then we can apply the concept of conditional expected values and any persistent difference between expectations and realization has to be considered as irrational. For our approach it becomes necessary to differentiate between the mathematical concept of the conditional expected values (or density functions) on the one hand and optimally formed decisions (plans, anticipations) which would be called actions in decision theory. It is the specific hypothesis of this study that empirically reported expectations (forecasts, plans) approximate the optimal decision. In contrast to the mathematical concept of the conditional expected value, these economic expectations incorporate risk attitude and consequences of eventual errors on the argument of the objective function. This, although not yet the Keynesian view of expectations, gives leeway for incorporating the determinants of 'expectations' mentioned by Keynes (1937) into decision relevant anticipations (plans, actions).

Thirdly, some conciliation is sought between the tendentially normative theory of firm under uncertainty, which knows price flexibility, the existence of future markets and mainly speculative inventory fluctuations, and the empirical literature, which takes certain price rigidities, involuntary inventory movements and rapid production changes for granted, trying to explain them frequently *ad hoc*. Business surveys and econometric results are used in selecting and constructing the models.

1.4 SOME CONCEPTS USED

The abbreviations used are summarized in appendix 1. The concepts used in general follow the literature but it may be useful here to repeat some of them.

The *decision variable* is that variable by which optimization is done. The *random variable* is the variable whose realization is not known *ex ante* and which is known *ex post* and under certainty (where we will assume it to take the value $X_0 = EX$). *Ex post flexibility* designates a situation in which the decision variable can be adjusted in a second step after the veil of uncertainty has lifted.

An *ex ante control* is a variable about which we have to decide before the veil of uncertainty is lifted, an *ex post control* can be chosen or is simply adjusted after the event. A *mode* is a situation characterized by the assumption about the *ex ante* control (a *p-mode*, for example, is behaviour in a model in which we assume that the price has to be set *ex ante*, in the *p-q-mode* both variables have to be decided *ex ante*).

As *disequilibrium models* we label models wherein production or demand differs or in which input installed and the input used differ. If there are *ex post* controls, part or all of the differences will optimally be bridged, so that there is no unsatisfied demand, unsold product or free capacity.

As *technological concavity* we label a marginal product concave in the decision variable, where the concavity stems from objective reasons (like technology, demand curve, etc.), in contrast to risk attitude.

We contrast the optimal decisions between *certainty* and *uncertainty*. Introduction of uncertainty is one subcase of *increasing uncertainty*. Usually the assumptions necessary for unambiguous results are stricter for the more general class of the problem. Propositions 1, 2 and 4 apply unmodified to the wider class of the problem, for proposition 3 the effect of increasing uncertainty usually depends on whether the optimal value will be in the range of $F(Y) \cong 0.5$. Since we argue that it will usually be in the 'smaller range', the effect of increasing uncertainty and of introduction of uncertainty will again tend in the same direction, though this may not be true under all circumstances.

We deal with *partial* decision models in the sense that one agent maximizes its utility in a certain given environment. Neither the behaviour

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of the other side of the market is changing nor the development of the market itself is modelled. For questions of industry equilibrium see Sheshinsky & Dreze (1976), for general equilibrium under uncertainty see Radner (1982). We model *passive situations* in the sense that firms do not learn actively about their environment and they cannot change the information set. For active or adaptive situations see Hey (1981) or Lippmann & McCall (1981).

Finally I want to thank all persons who helped me through the many periods of uncertainty I passed in writing this book. Among the academics I would like to thank first of all my teacher Erich Streissler of the University of Vienna for his persistently encouraging comment, then my colleagues Michael Winkler, Michael Wüger and Michael Pfaffermayr for help and control when problems threatened to outgrow my finite set of mathematical knowledge.⁸

NOTES

- 1 Cf. Aiginger 1977, 1979, 1981a,b, 1983.
- 2 Neglect of future markets can also be justified on the grounds that uncertainty in particular inhibits everyone from committing themselves for too long, and that future markets (for most goods) do not exist (Rothschild 1981, p. 24).
- 3 First edition: 1921.
- 4 i.i.d. (identically, independently distributed) means that the same distribution function is assumed for each period, irrespective of any (the most recent) realizations.
- 5 Thus Leland (1972) claims that the quantity setting the monopolist's production decision does not change through uncertainty; Nickell (1978) arrives at the opposite view without explaining the contradiction to Leland's finding. Hymans (1966) comes to the conclusion that less is produced in the competition model under uncertainty than under certainty, whereas we find it asserted in the bulk of the literature that output is the same in each case.
- 6 To substantiate the claim that rational expectations should yield on average correct forecasts, as well as that most authors disregard influences on optimal decision under uncertainty that go beyond the risk aversion, we may quote Arrow (1978, p. 159): 'A moderate version of the rational expectations' hypothesis is that the anticipated price equals the expected price. A stronger version is that the economic agent knows that the price is a random variable and uses in his decisions the true distribution. (The stronger hypothesis is significant if the agent is a risk averter, so that his decisions are not determined merely by knowledge of the expected value).' The quotation discloses the neglect of all forms of technological concavity, of disequilibrium costs and asymmetries in *ex post* flexibility. Neglect of the first component is especially astonishing since a general condition concerning the effect of uncertainty (U_{YXX} , cf. equation 3 in chapter 4) has been known since Rothschild & Stiglitz (1971). From this it may be seen that it is not the utility function alone which decides on the optimal action.
- 7 Cf. the neglect in the excellent summary studies of Hey (1979, 1981) or in the article by Lippmann & McCall (1981).
- 8 Thanks to to Fred Prager for translating the book, to Dagmar Guttman, Elisabeth Lebar, Traude Novak, Maria Seidl and Getrude Wenz for collecting the data, rearranging the calculations, drawing up the tables and for typing, checking, retyping etc. Manfred Nermuth, Gerhard Orosel, Kurt Rothschild, Gunther Tichy and Georg Winckler supplied me with critical comments in earlier stages of the work.

Part II

Theoretical part

2 Interpreting Reported Expectations as Optimal Decisions (Actions) under Uncertainty

The term 'expectations' is extensively used by firms, politicians and economists. This is done often without specifying the underlying concept of 'expectations'. I will use the term within inverted commas whenever I refer to expectations without making explicit the definition used.

In particular, there seems to be an unbridgeable gap between the 'expectations' reported by firms as sales forecast, as investment anticipations or price forecasts on the one side, and the term 'expectations' used by macroeconomists if they refer to conditional expected values (a concept known as Rational Expectations' Hypothesis). We want to include in the first group of 'expectations' firms' expectations, but also consumers' expectations and experts' forecasts for macroeconomic variables and label them 'reported expectations'; while we will use the term *mathematical expectation* for concepts like expected values or density functions.

We will show in section 2.1 that there are striking divergencies between reported expectations and implications of mathematical concepts, among them that reported expectations are biased as seen from the viewpoint of the most popular operationalization of the Rational Expectations' Hypothesis. The bias is discussed in section 2.2. We will then develop the hypothesis that reported expectations can be considered as (approximations) to *economically rational expectations* which deviate from pure mathematical concepts insofar as they include the consequences of potentially incorrect expectations (2.3). The merits and disadvantages of this concept of economically rational expectations and the role of uncertainty theory to demonstrate that this concept may not only be a feasible but an important one, are discussed in section 2.4.

Table 2.1 Mean and unbiasedness of thirty-nine series of reported expectations testing for the identity (H_0) of expected and actual changes (t -test)

	H_0 rejected 95% (99%)	H_0 non-rejected	Mean of expectations		Among the rejections of H_0	
			lower than the mean of realizations	higher	lower	higher
All data	15 (13)	24 (26)	29	10	14	1
Quantitative business surveys	10 (9)	6 (7)	14	2	9	1
Qualitative business surveys	0 (0)	7 (7)	3	4	0	0
Consumer surveys	3 (3)	0 (0)	3	0	3	0
Experts	2 (1)	11 (12)	9	4	2	0
Testing for unbiasedness						
$H_0: (k, b) = (0, 1)$ in $a_t = k + be_t$; F -test						
All data	21 (16)	18 (23)				
Quantitative business surveys	14 (11)	2 (5)				
Qualitative business surveys	2 (1)	5 (6)				
Consumer surveys	3 (3)	0 (0)				
Experts	2 (1)	11 (12)				

Source: Aiginger (1981b)
 a , e actual change (e.g. of sales), expected changes.

2.1 THE EMPIRICAL PHENOMENON OF BIASED REPORTED EXPECTATIONS

Data on business and consumer expectations are available for many countries, variables and business sectors. One of the striking results of most empirical investigation is that reported expected changes (of sales, output, inflation) are consistently (for long periods) lower than actual changes. This difference is found for medians as well as for means; it is even present – albeit smaller – in periods of decelerating growth. It may be demonstrated on the aggregate and on the micro level. For an overview on the aggregate level see Aiginger (1979, 1981a,b), on the level of individual answers Figlewsky & Wachtel (1981), Zarnowitz (1981), Nerlove (1986). Zimmermann (1984), Kawasaki & Zimmermann (1984). The underestimation tendency is especially strong for entrepreneurial expectations. Macroeconomic forecasts¹ underestimated real growth in the 1960s, but not in the 1970s (though the really surprising fact is that growth forecasts were not too optimistic in this period of negative shocks). Experts underestimate actual inflation according to the famous ‘Livingston survey’ for the US, the same seems to be true for Michigan’s consumer expectations (for these data of the Consumer Research Institute the final assessment depends on the quantification procedure used), and for Austrian data on expected inflation (Breuss & Wüger 1986). Inflation forecasts by macroeconomic forecasting institutions seem to escape the spell of downward-biased forecasts (Aiginger 1979, Neumann & Buscher 1980, Kirchgässner 1982).

Table 2.1 shows the bias for a sample of thirty-nine variables grouped into quantitative business expectations (about sales, investment, exports), into qualitative business data (about production, order and price trends), consumer expectations and experts’ forecast. The average expected change is smaller for twenty-nine variables out of thirty-nine as compared with actual change. In testing for unbiasedness in the usual sense of a zero constant and a unity regression coefficient, twenty-one variables failed to pass the test. Among the subset of the quantitative business expectations unbiasedness had to be rejected for fourteen out of fifteen variables at the 95% level of significance. Table 2.1 replicates results from Aiginger (1981b); the data mainly ended in 1975.

Tables 2.2 and 2.3 present data for Japan and the US which demonstrate that the bias did persist in the period of slow growth since the mid-1970s. For all four variables expected changes in Japan are lower than the actual ones also in the period 1976–85; in two cases the bias is statistically significant even in this short slow growth period 1976–85. For the US, Livingston data show that experts continued to underestimate inflation and industrial output.

Table 2.2 Expected and actual production (sales, exports, investment) in the Japanese manufacturing industry

	1963-75		1976-85		1963-85	
	Mean	SD	Mean	SD	Mean	SD
Production						
expected change	2.92	0.83	1.52	0.86	2.33	1.09
actual change	3.54	2.90	1.89	2.24	2.84	2.77
significance of difference	—	**	—	**	—	**
Sales						
expected change	3.06	0.95	1.67	0.93	2.47	1.18
actual change	3.55	2.40	1.92	2.42	2.87	2.59
significance of difference	—	**	—	**	—	**
Exports						
expected change	2.96	2.87	0.77	1.53	2.03	2.61
actual change	5.38	6.43	2.66	3.56	4.24	5.59
significance of difference	**	**	**	**	**	**
Investment						
expected change	0.45	3.56	-5.12	5.44	-1.90	5.23
actual change	2.79	6.93	1.59	7.25	2.28	7.05
significance of difference	*	**	**	**	**	*

SD = standard deviation. *Source:* Bank of Japan.

Table 2.3 Expected and realized changes (annualized) for inflation and industrial production (1961-85) in the US

Livingston	Consumer price		Industrial production	
	mean	SD	mean	SD
6-month forecast	4.34	2.65	2.49	3.99
12-month forecast	4.50	2.56	3.28	2.66
actual change	5.40	3.56	4.16	5.88
Significance of differences for forecast as compared to actual change				
6 month	*		**	
12 month	—		—	

Source: Livingston Survey, USA. *(**) denote 95(99%) level of significance

2.2 BIASEDNESS AND RATIONALITY

Biasedness of reported expectations are in conflict with at least that operationalization of the 'Rational Expectations' Hypothesis' (REH) which dominates macroeconomic literature (and even more economic policy literature).

The basic idea of the REH approach as proposed by Muth (1961) is that information is an economic good, which should be used economically in

the process of the formation of expectations, and secondly that expectations depend on the whole structure of the economic system and therefore cannot be independent from economic policy. In the following application of this REH in economic models and especially for evaluating the effects of economic policy in a world of 'rational expectations', the concept was nearly without exception cast into the specialized version of the conditional expected value. This specialized version of the wider (and nearly irrefutable) notion was suggested by examples in the article by Muth himself, though Muth did not forget to stress that he had switched to a special form² of the hypothesis implying a quadratic objective function (and allowing a linear expectation operator). This specialized version allowed a whole battery of empirical tests on the 'rationality' of reported expectations, which start from the presumption that errors in the expectations should on average be zero (more exactly, the expected error should be zero, a tendency which in the long run should imply that *ex post* errors level out too).

The fact that Muth wanted to propose a notion of rationality which did not hinge on the linearity of models is evident since he was a member of a research group at the Carnegie Mellon University which had intensively studied the applicability and advantages of certainty equivalents (Holt *et al.* 1960). However, economists making use of his hypothesis clung to the specialized form of the conditional expected value. This concept influenced economic policy in the 1970s in a very profound way leading to the so-called 'Policy Inefficiency Theorem': together with its twin hypothesis that economic agents act in response to differences between expected and realized values, rational expectations guarantee that systematic economic policy (like monetary policy) does not matter. The most prominent articles of this 'rational expectation revolution' are Lucas (1972), Sargent (1976), Sargent & Wallace (1976), and McCallum (1976, 1978, 1979). For overviews on this literature see Mishkin (1983), Sheffrin (1983), or Carter & Maddock (1984). There had been objections against the inefficiency theorem, focusing on the twin hypothesis, on the informational assumptions of the REH, it was shown that overlapping or divergent expectations between individuals, etc., would destroy the strong implications of the REH, but there had been no critique that that same inefficiency result crucially depended on the assumption of linear models. Shiller (1978) conjectured this in a survey without finding the dependence on the linearity extremely restrictive, Lucas (1978) used a concept in which the whole subjective probability function of the agents had to match the objective probability function and theoretical studies tried to find whether a unique solution could be found for non-linear, rational-expectation models. But most applications of the concept for economic policy as well as all tests on the 'rationality' of reported forecasts implicitly or explicitly assumed linearity. We will therefore refer to this specialized version of the hypothesis which moulds 'rational expectations' into the expected

Table 2.4 Synopsis of alternative concepts of 'expectations'

Concept	Definition	Examples
'Expectations'	unspecified concept, implicitly assumed to be the only sensible concept by the individual applicant	e.g. politicians declare investment activity to be hampered by 'expectations' economists want to specify the influence of economic policy on the formation of 'expectations' importance of inflationary 'expectations'
Mathematical expectations	density function of a random variable expected value or conditional expected value (as a one-point approximation, e.g. MAREH, equation 2.1)	density function for demand or sales; most likely demand or sales (where expected demand or sales are independent from consequences)
Economically rational expectations	optimal decision incorporating the expected utility of the decision using a probability assessment, the set of consequences and a utility function (equation 2.3)	sales forecast underlying the production; expected monetary growth (inflation) used for evaluating wage offers, forecast published by forecaster
Psychological (irrational) expectations	subjective assessment (evaluation, mood) whose formation has other than economic reasons	waves of optimism and despair contagious processes according to Jöhr or Pigou the incalculable part of the decision (sudden changes in long-run expectations according to Keynes)

Hypothesis in this book: the reported expectations (plans or forecasts supplied in entrepreneurial surveys, inflation expectations reported in consumer surveys, published macroforecasts) are approximations to economically rational expectations.

conditional value as the 'Mainstream Rational Expectation Hypothesis' (MAREH). It implies that 'one need only apply a very weak form of the Rational Expectations' Hypothesis to infer that inflation cannot be under- or overestimated year after year' (Poole 1976, p. 465). Biased expectation, in the sense of a persistent difference between expected and realized values, as well as biasedness in the sense that regressing actual data on expectations does not yield a zero constant and a unity regression coefficient, is in conflict with the MAREH.

One reaction to the 'bias' of reported expectation is to declare them 'unreliable', invoking 'errors of measurement', 'sampling biases', (McCallum 1976, Lahiri 1976, etc.). This does not seem a good strategy, since there is evidence that reported expectations possess the same or even (in period of structural changes) better forecasting performance than econometric investment function or consumption functions or autoregressive proxies, and they add to the forecasting performance of macroeconomic models.³ We know that production expectations lead actual production (or at least published data about production), that inflationary expectations outperform autoregressive proxies, etc. Predictive ability, for turning points and cyclical developments on the one hand and sampling errors and irrational biases on the other, is an improbable combination.

2.3 INTERPRETING REPORTED EXPECTATIONS AS OPTIMAL FORECASTS UNDER UNCERTAINTY

The alternative proposed in this book is to interpret reported expectations as optimal forecasts or as actions in the terminology of decision theory, including an evaluation of the consequences (gains or losses) of 'wrong' reported expectations. In its most general version we can use the well-known concept of expected utility maximization, where we interpret the reported expectation to be the decision variable of this maximization. As an intermediate step we propose a concept of Loss-Evaluating Rational Expectations (LEREH), which is a specialized version of expected utility maximization in which the source of the bias of reported expectations is made evident.

The Mainstream Rational Expectation Hypothesis (MAREH) is described by formula 2.1. We know a probability function of the variable x and use the information available up to the end of $t-1$. The use of the conditional expected value as a one-point representation, Ex , of the random variable can be justified by the assumption of a symmetric quadratic loss function in case of errors.

Standard operationalization of the REH (MAREH):

$$\hat{x} = \int_{-\infty}^{\infty} xf(x|I_{-1})dx = Ex|I_{-1}. \quad (2.1)$$

Considering the possibility of a more general loss function $L(y, x)$ where losses depend on the action, y , and the random variable, x , yields a concept of 'loss-evaluation expectations'. An optimal choice of y depends now on the property of the loss function. A special case of Loss-Evaluating Expectation is given if the decision variable and random variable coincide (see equation 2.2a). Optimal expectation (for example, the optimal sales forecast \hat{x}) now depends on losses accruing from differences in the actual changes and expected changes x^e). If the losses due to a positive difference and a negative difference are identical, we can return to the mainstream version. If larger than actual expected sales yield a smaller loss than the contrary error of the same size it is rational to report (act on) a lower expected value. Economically rational expectation has in this case rationally and persistently to be lower than the mathematical expectation.

Loss -Evaluating Rational Expectations (LEREH) (where y is a decision variable and x a random variable):

$$\hat{Y} \doteq \min_y \int_{-\infty}^{\infty} L(x, y) f(x|I_{-1}) dx = \min_y \int_x EL(x, y). \quad (2.2)$$

Special case where $x = y$:

$$\hat{x} \doteq \min_{x^e} \int_{-\infty}^{\infty} L(x, x^e) f(x|I_{-1}) dx = \min_{x^e} \int_x EL(x, x^e), \quad (2.2a)$$

where the sign \doteq means 'is derived from', $L(x, x^e)$ means loss in case of plan (reported expectation) x^e and the realization x , $L(x, y)$ means loss of the decision y and the realization x , $f(x|I_{-1})$ means density function of x (conditional on I_{-1}).

The most general version of decision-making under uncertainty is to cast the problem into the expected-utility framework. Expected utility depends on the utility function $U(Z)$, the probability function, $f(x)$, and the consequences $Z(y, x)$. Expected utility is maximized, and we interpret the optimal value of the decision variable as that figure aimed at in the firm's planning process and also reported in surveys and forecasts (reported expectations). For example, the optimal sales plan, \hat{y} , is chosen in a world of price uncertainty ($x = \text{price}$).

General concept for economically rational expectations (LEREH):

$$\hat{y} = \max_y \int_{-\infty}^{\infty} U[Z(x, y)] f(x|I_{-1}) dx = \max_y \int_x E U[Z(x, y)], \quad (2.3)$$

where $U(Z)$ is utility function and \hat{y} is optimal decision (economically rational expectation).

To interpret reported expectations as approximations to optimal decisions under uncertainty is by no means an innocent concept. One of

the main premises of decision theory requires a distinction between 'expectations' and 'consequences'. Expectations as defined by decision theory are a mathematical or statistical concept (for example a density function or as a one-point approximation of its expected value), these expectations have to be independent from individual preferences and consequences. The assumption that reported expectations may be economically rational because they incorporate intentionally the (maybe asymmetric) consequences of wrong 'expectations' is equivalent to interpreting 'expectations' as optimal actions (decisions) in the terminology of decision theory. The concept of mathematical expectations as used in the decision theory and the proposal of this book to interpret reported expectations as deliberately incorporating the consequences of expectational errors should be distinguished carefully to save confusion. We want to label our concept as 'economically rational expectations' to stress that reported expectations diverge from mathematical ones due to economic reasons (cost and demand conditions, shape of utility function, etc.), and not due to irrationality or psychology.

We have to admit that economic theory today nearly without exception identifies 'expectations' with the concept of mathematical expectations.⁴ In more technical and more theoretical work expectations are usually modelled as density (probability) functions, in more applied work mainly expected values are used. As mentioned already, models which try to incorporate the impact of expectations on economic policy nearly without exception use that operationalization of the REH concept in the way that 'expectations' are conditional expected values.

This is probably not the kind of 'expectations', which was focused on as economists started to worry about 'expectations'. Pigou (1927) wanted to demonstrate how waves of optimism and pessimism were able to generate business cycles; Keynes (1936) stressed the importance of expectations for the investment process and wanted to refer to the incalculable part of the decision; Jöhr's interpretation of cycles as 'contagious diseases' can hardly be represented by simple mathematical concepts. Muth wanted to develop a broader concept of rationality, but conceded that real-world expectations would diverge from any concept of rationality – explicitly he mentions over- or under-assessment of recent information (Muth 1961, p. 321).

However, neither is it relevant which concept of expectations economic theory used at its beginning, nor which dominates today, but which phenomenon we want to deal with. If we want to describe a situation in which a casual and personally hardly interested person is asked to make a forecast, it seems plausible to accept mathematical concepts of expectations which abstract from consequences of wrong forecasts. But if we want to describe the process in which economic agents derive those decisions on which they have to act personally with high stakes at issue, or the forecast of experts on which their prestige and future incomes will depend, we

cannot abstract from the incorporation of (potentially asymmetric) consequences. There is only the alternative to model a one-step decision or a two-step decision process. A one-step decision would purport that the economic agent forms an economically based decision which jointly assesses mathematical expectations and loss considerations. A two-step procedure would involve economic agents first trying to find a mathematical expectation (usually a density function), then in the second step considering the economic consequences of errors before making the decision. The two-step process is the analytically cleaner procedure. But there are not too many indicators that the two steps are separated explicitly in real-world decisions. Economic models with rational expectations do not use a two-step process either (only the first step is modelled), usually firms make only a single forecast for future investment and sales. Surveys do not specify which sales forecast they would like to know in cases where two (or more) forecasts are made. Unbiasedness is considered one of the least important criteria for accuracy of macro forecasts (implying quadratic symmetric losses). Casual evidence therefore supports that firms do not distinguish between 'mathematically expected sales' and 'sales forecast on which employment and production decisions are based', nor do macro forecasters in practice distinguish between things they believe and things they forecast because they are optimal for the economy or for their own prestige. From a normative point of view, agents should use a two-step process, but even then published sales expectations or forecasts will be more likely 'economically rational forecasts' than mathematical expectations.⁵

To sum up, the hypothesis for understanding 'expectations' in this book will be that *reported expectations* (plans or forecasts supplied by firms in surveys, inflation or income expectations reported in consumer surveys, macroeconomic forecasts published by institutions) *are approximations to economically rational expectations*. These economically rational expectations are actions (decisions) in the terminology of decision theory incorporating mathematical expectations, loss and utility considerations. Reported expectations therefore can be modelled with the usual expected utility maximization procedure (see equation 2.3).

An important implication of this hypothesis is that reported expectations may diverge from realizations even in the long run (on average) in the case of asymmetric consequences (for errors of opposite sign but the same size), or more generally in the case of non-linear models. In particular, the downward bias empirically demonstrated in section 2.1. is economically rational if the negative consequences of a 'too-low' value of the decision variable are less *severe* than those of a 'too-high' value.

What could be the reasons for such an asymmetry? The models presented in chapters 3–8 try to find out why biases are not only feasible, but are very likely and usually work in the 'downward' direction. The feasibility alone would not be very surprising, since it is well known in statistical

literature that expected values are a fair representation in case of linear models only. Unbiasedness is a valid measure of forecasting performance only in the case of symmetric losses. This again is well known in statistics, though the knowledge had been temporarily forgotten in the 'policy ineffectiveness literature' which based strong conclusions on linearity without evidence (see appendix 3 for an illustration).

2.4 WHERE TO GO FROM HERE

We apply the theory of the firm under uncertainty (chapter 5), inventory theory (chapter 6) and models on input decisions (chapter 7) to find optimal values of output, prices, inventories, etc., under uncertainty. We then compare these optimal values with their counterpart in the world of certainty. In cases where the unique optimal values under certainty do not exist (for example, under competition with linear costs), the optimal decision will be compared with some expected value (for example, with expected demand).

The eventual result that the optimal value under uncertainty is lower than under certainty is related to, but not identical with, the finding that reported expectation (expected sales as reported in a survey) will optimally be lower than actual sales. Strictly speaking the models would imply that the expected and actual value of a decision variable would have to be identical, since what has to be decided *ex ante* cannot be changed *ex post*. Most models separate variables into some which are *ex ante* control variables and cannot be changed *ex post* and some which are *ex post* control variables for which it is not necessary to make an *ex ante* assessment.

The persistent biases of surveys on the aggregate level and the much more volatile relations between plans and actual outcome on the micro level 'indicate'⁶ that in practice most variables supposed to be *ex ante* control variables (at least for a part of the firms) like production, investment, employment, can be altered during the period in which uncertainty is gradually lifted. A complete fixedness for example of production would imply – together with sticky prices – extreme imbalances. Therefore it seems to be realistic to assume a certain degree of *ex post* flexibility in the sense of partial and costly *ex post* revisions of the decision variables.

If the *ex post* adjustment of the decision variable is feasible it has to be incorporated into the optimal decision. We develop such models in chapter 8.

In the case of *ex post* flexibility, the relation between biased reported expectations and the impact of uncertainty is quite straightforward. Uncertainty leads to a cautious (low) preliminary choice of the decision variable, later on this can be revised (upwards). Reported values of the decision variable (in a survey or a firm's plan) and later 'realized' values

have to be different. For the majority of the models the relation is not so straightforward; maybe from a purist's point of view one could even insist that the impact of uncertainty on the optimal decision and bias are totally unrelated problems. We think, however, that the demonstration that uncertainty leads to lower values for decision variables tends to 'open space' for later upward revisions at least for aggregate data. This is evident for disequilibrium models, where there is no *ex post* control to close the gaps between production and demand.

Our models are intentionally partial models which specify the optimal decision of firms. We can imagine that the market equilibrium⁷ is reached in one of the following ways: the first may be that there exist two sectors of firms in the economy, the first being large firms which have to make early commitments (as modelled in the theory of firms) and the second consisting of small firms, which are able to close the discrepancies due to their flexibility. A second way would be that though a variable is an *ex ante* control in principle, fine-tuning of the decision variable (not modelled in theoretical models) is feasible to close a small part of the discrepancy.⁸ In either of these cases the reported expectation may differ a little – though in a systematic way – from actual values, in the first case in the aggregate only, in the second for each optimizing firm. The greater and the less ambiguous the impact of uncertainty on the firms, the greater the downward bias will be for a given 'inexactness' of the theoretical model.

In surveys on sales, prices or investments firms are asked for their estimates whether these variables are *ex ante* or *ex post* controls. We know from the theory of firms that different modes may be optimal under uncertainty depending on the cost structure. For example, the monopolist can choose the price and use demand as an *ex post* control, he can choose quantity and use demand as an *ex post* control or he can fix both *ex ante*. Costs will decide (see Lim 1980) which is better for expected profits, the optimal and implied values for the individual variables are not identical under these three modes. Therefore it can and will happen that a given aggregate of reported expectations will be a mixture of reports about decision variables, expectation about the value of an *ex post* control and perhaps sometimes of variables for which fine-tuning of preliminary decisions is possible. If theory predicts that the optimal (preliminary) decision under uncertainty will be lower than under certainty under a lot of circumstances, the aggregate of reported expectations (as formed in an optimal or suboptimal procedure, considered as preliminary or fixed, etc.), will tend to be lower than the later outcome.

NOTES

1 Cf. Mincer & Zarnowitz (1969), Zarnowitz (1981), Aiginger (1983).

2 Cf. Muth (1961), n. 4 on p. 317.

- 3 Cf. Okun (1960), Friend & Taubmann (1964), Friend & Thomas (1970), Fair (1971), Kuhbier & Sauer (1983).
- 4 Pertinent literature may at a stretch concede a distinction between mathematical and psychological expectations (see Poole 1976, p. 465). As distinct from that implied insinuation of 'irrational' expectations, the present study considers the deviation of measured expectations from REH economically well founded ('economic' expectations in contrast both to 'mathematical' and 'irrational' expectations).
- 5 Interestingly enough, protagonists of MAREH argue precisely against empirically measured expectations with the argument that these, contrary to implicit expectations (as for instance resulting from actually pertaining nominal interest, if the latter is interpreted as the sum of a constant real rate of interest and expected inflation) are not the true action relevant expectations, because 'reporting' of expectations in a field survey does not involve consequences. (Cf. Pyle (1972), Pesando (1975), McCallum (1976).) Against this we argue the production, sales and investment expectations are precisely the entrepreneurial plans that are the basis of the real production process as carried out by management, taking into account uncertainty, production technology and market development.
- 6 We choose the verb 'indicate' because from the discrepancies visible on the aggregated level (and even bigger ones on the micro level) all that can be deduced is that there are either 'involuntary changes of plan' (e.g. caused by breakdowns of machinery) or deliberately introduced revisions. It may also be that production is no *ex ante* control at all.
- 7 Examples of problems of market equilibrium under uncertainty may be found in the following studies (and the literature quoted there): Sheshinsky & Dreze (1976), Drazen (1980), Hey (1981, p. 181ff.), Radner (1982).
- 8 Finally it might also be possible that some firms use price *and* output as *ex post* control (in part as a decision variable, partly as a performance target). This would accord with the certainty situation – as shown in the discussion of the outcomes by Oi (1961) (cf. Nelson 1961, Hey 1979, Pleeter & Horowitz 1974). During a period of 'uncertain demand' it is plausible that firms of different size may wait for different periods before committing themselves about their output. Large firms are probably inclined to the concept of *ex ante* control via at least one variable, whereas smaller firms may tend to the situation of *ex post* control by means of all variables.

3 Expected Utility Maximization and its Alternatives

Economic theory of uncertainty is dominated by the assumption that economic agents behave as if they were maximizing expected utility.¹ In this book we follow along this tradition of mainstream theory. The first reason for this is that this flexible and elegant framework allows one to model an enormous variety of situations in a consistent way. The second is that it is much harder to prove the notion 'that agents should behave cautiously under risks' within this framework than under some less elegant alternatives.

Concerning Knight's dichotomization between 'uncertainty' as the situation in which agents do not possess a probability function over the random variable, and 'risk' as the situation where they do, we refer in the following to the latter.² Again the formal elegance and the fact that the environment of risk seems to be the stronger test for our hypothesis is the reason for this. In cases of total ignorance minimax rules or the Hurwicz rule are criteria which lead to rather pessimistic decisions.

The same is true for the mean variance criteria which dominate portfolio theory³ and tend to suggest optimal decision parameters below the respective certainty values. The mean variance criterion, however, is consistent with expected utility maximization at least under restrictive circumstances.⁴

Different versions of 'safety-first principles' minimize the probability of losses,⁵ the probability that loss exceeds some critical value or some similar criterion. Arzac (1976) proposes four different criteria (applying them to competition under price uncertainty) and finds that production is less than (or the same as) that under certainty. Some versions of the safety-first principle are consistent with expected utility maximization, implying risk-averse utility functions.

Hey (1980) argues that there is no *a priori* reason why we should define as risk-neutral agents indifferent between a certain prospect and an uncertain one with the same (arithmetic) mean. If we define as risk-neutral an agent indifferent to a prospect with the same geometric mean we get a logarithmic utility function. Agents risk-neutral in this sense should now behave like risk-averse agents in the standard theory.

Kahneman & Tversky (1979) base their critique of the standard theory on empirically revealed preferences between prospects which are inconsistent with expected utility theory in at least three respects. The 'certainty effect' means that people underweight outcomes that are obtained with certainty. This tendency contributes to risk aversion in choices involving sure gains and to risk-seeking in choices involving sure losses. The 'isolation effect' characterizes the behaviour that people generally discard components that are shared by all prospects under consideration. This tendency leads to inconsistent preferences when the same choice is presented in different forms. The 'reflection effect' means that people behave risk-averse in the positive domain but risk-seeking if they have to choose between alternative losses. Kahneman & Tversky develop an alternative theory, called prospect theory, in which value is assigned to gains and losses rather than to final assets, and probabilities are replaced by decision weights (which may not add up to one). This value function is normally concave for gains, commonly concave for losses, and is generally steeper for losses than for gains.

'Regret theory' shares the intention of prospect theory to explain the empirical violations of the expected utility maximization, but purports to do this in a much simpler way (Loomes & Sugdan 1982). In contrast to 'choiceless utility' which is experienced from an income an agent experiences without choice, people experience sensations called regret and rejoicing if they can choose among alternatives. They experience regret if they know with hindsight that they could have chosen a better action for the realized state of nature, and they experience rejoicing if they have made a good choice. Formally, people are maximizing the expected value of a modified utility function depending on the choiceless utility of 'what is' and of sensations of 'what might have been'.⁶ As one of the many extensions of regret theory we want to refer to Bell (1983) who calculates a risk premium for decision regret and a wider definition of risk aversion, now combining two components: decreasing value and regret aversion.

Another group of papers were intended to formalize 'that individuals generally avoid situations which offer the potential for substantial gains but which also leave them even slightly vulnerable to losses below some critical value' (Menezess *et al.* 1980, p. 921). This behaviour labelled 'aversion to downside risk' or to 'below-target returns' is related to prospect theory insofar as it uses a reference point to translate monetary outcome into gains and losses, and that the utility (or value) function is fundamentally different for gains and losses. It is further related to the safety-first criteria and to decision criteria using higher moments (see again Menezess *et al.* 1980). One distribution has more downside risk than another if it can be obtained from the other by shifting dispersion from the right to the left without changing mean and variance. Downside risk aversion can be compatible with expected utility theory if it is defined as

a presumption about the third derivative of the utility function. In principle, risk-averse and risk-seeking individuals can have utility function with a negative third derivative of their utility function. One of the empirically important rationales for downside risk aversion is the possibility of ruinous losses. Laughunn *et al.* (1980) investigated the risk preferences for below-target returns of 224 managers and reported them to be risk-seeking as long as no ruinous losses were included, but the majority switched to risk aversion for below-target returns after ruinous losses were included. Downside risk aversion tends to bias downwardly optimal decision as compared with certainty – for example, the competitive firm under price uncertainty produces less under uncertainty and downside risk aversion, (see Stewart 1982, p. 146).

Another attack on the descriptive value of expected utility theory stresses the cognitive limits of agents. Since most economic decisions are extremely complicated, simultaneous maximization of all relevant variables exceeds the cognitive limits of the decision maker and people usually restrict their efforts to reach a satisfactory level of targets, maybe in a sequential decision process (Simon 1955, 1978). Radner (1975) has developed a formalization for satisficing behaviour. Shackle (1955) constructed a model where people base their decision on focus values (which describe one favourable and one unfavourable scenario) and on potential surprises; Pye (1978) proposed one in which flexibility and robustness are important.

Instead of violating the premises of expected utility maximization Machina (1982) tried to incorporate the conflicting empirical evidence by generalizing the Neumann-Morgenstern theory. Machina shows that the results of the expected utility hypothesis can be derived if we substitute the independence axiom by a more general one ('smoothness of preferences over alternative probability functions'). Machina's 'generalized expected utility theory' is no more in conflict with the Allais or the Petersburg paradoxa, it is able to explain the coexistence of lotteries and insurances and downside risk aversion.

The great divide of uncertainty theory into models dealing with risk and models dealing with uncertainty proper has been blurred already by the contribution of Simon, Shackle, Kahneman, Tversky and others; further compromises are offered by models with partial information where only rough guesses of the relevant probabilities are available or at least the ranking of the probabilities of the states of nature is available (see DeFinetti 1937, De Groot 1970).

Nevertheless the great divide of decision theory has found its parallel in economic theory. Neoclassical economists rely on expected utility theory modelling the world of uncertainty similar to that of certainty but substituting a known variable by an uncertain one (sometimes simply by the help of an expected value). Keynes and the Post-Keynesians on the one hand purport that economic agents will behave qualitatively different under uncertainty. Economic uncertainty has to be characterized as singular or at least unrepetitive constellation (Rothschild 1981, p. 107), in which agents

cannot assess probabilities for the outcomes, sometimes they do not even know the relevant alternatives, or as Keynes put it, 'we simply don't know' (Keynes 1937, p. 113f). As a consequence of uncertainty proper, agents regress to simple rule of thumb or conventions, they behave in a conservative way reacting only to dramatic changes in the environment.⁷ Some specific reactions to uncertainty proper are mentioned in the following (Rothschild 1981):

- the importance of liquid assets is increased;
- conservative price-setting and price and wage rigidities become rational even in the absence of monopolistic or oligopolistic markets;
- economic agents will refrain from long-term commitments;
- firms will prefer flexible production techniques; and
- disequilibria will be a widespread phenomenon.

Though we use the technique of expected utility maximization in this book we will try to bridge the divide in economic theory a little. We will respect the arguments of the Post-Keynesians in the selection of the models and in the upshot we will get some results which are Keynesian in spirit.

Proponents of the Keynesian school never tire of stressing that price rigidity will be a rational strategy under uncertainty; we follow this presumption insofar as we give more room to models with price rigidity than other monographs do (for example we do not dismiss the competitive model with demand uncertainty, see section 5.1.5, which is in conflict with the taste of neoclassical economists). We allow a monopoly model without any *ex post* control though under most circumstances expected profit is lower under this mode than under alternatives. We construct a model where the preliminary value of the decision variable can be adjusted *ex post*. The lower the cost of adjustments, the higher are expected profits in this mode.⁸ Keynesian arguments suggest that we should not dismiss these models, hence we apply the formal apparatus of expected utility maximization on these as well as on the more neoclassical models.

We are able to show that optimal decision will differ under uncertainty from certainty partly depending on the third derivative of the objective function, partly due to the very existence of uncertainty. In the summary of the findings of the theoretical models (chapter 9) we raise the question whether we should try to structure the results into a group where uncertainty seems to be of less importance and one where it changes optimal result to a greater extent (section 9.2). We will label the first group 'petty uncertainty' since there differences between certainty and uncertainty are relevant only under circumstances about which we do not know much empirically and since the *ex post* control allows one to mitigate the effects of uncertainty. The second group will be labelled 'severe uncertainty' since uncertainty changes optimal decisions independent of specific ranges of a parameter or the curvature of the objective function, and there is no

ex post control. Though 'severe uncertainty' is related to uncertainty proper and to the Keynesian view, and though 'petty uncertainty' is related to risk and to conventional neoclassical models, in both concepts we apply expected utility maximization. It is the way we construct the model (what we do consider as endogenous, whether there is an *ex post* control, etc.), which differs, not whether probabilities can be assessed or not.⁹

To sum up we can see that there are a lot of problems with the dominant paradigm of decision theory, namely with the expected utility theory.¹⁰ Empirical experiments have revealed behaviour inconsistent with some of the premises of expected utility theory indicating that the theory does not work on the empirical level. They cannot however prove that people should not behave according to expected utility theory from the normative point of view. Alternative theories like prospect theory, regret theory, aversion to downside risk, generalized expected utility theory, etc., have been developed, other authors suggesting that people are not really maximizing or that people cannot assess probabilities. We nevertheless will apply expected utility theory in the following chapters, since most of the alternatives are not operational or convincing in general, while they have merits to explain important specific issues. We furthermore will try to show that many stylized facts which can be seen in the real world (and are used by opponents of expected utility theory as evidence against this theory) can be explained as rational by the means of expected utility theory. It is often the way the world is modelled (the objective function, its constraints, which variables are exogenous, etc.), which matters, not the method used to solve the models.

NOTES

- 1 For an overview on this theory and its axioms see Luce & Raiffa (1957) or Hey (1979); for a critique see Kahneman & Tversky (1979), Allais & Hagen (1979), Schoemaker (1982).
- 2 At the same time we assume a probability density function to be continuous, symmetric and differentiable (thus there being a distribution function which, by definition, assumes values between zero and one). The symmetry of the density function is not required for the bulk of the outcomes; it acquires importance where the decision under uncertainty is compared with the expected value of demand (cf. section 5.2.4 and chapter 6); occasionally it facilitates the derivation of the result. Given asymmetrical density functions the results would only make comparison of optimal action with the median of the distribution. Assumption of symmetrical density can be justified by assuming that a firm's demand consists of a relatively large 'certain' and a relatively small 'uncertain' part, and models are only meant to model the latter. For total demand, symmetrical distributions would not be very likely (e.g. because zero, anyway as a rule, would constitute an absolute limit for quantities in demand, and (conversely) very large quantities – though unlikely – would not be impossible).
- 3 Cf. Markowitz (1952, 1959), Tobin (1958). For higher moments see Allais & Hagen (1979).

- 4 There is compatibility if the utility function is quadratic or when the yields of each and all investment outlets are normally attributed.
- 5 Roy (1952).
- 6 See Fishburn (1977), Machina (1982), Loomes & Sugdan (1982).
- 7 Keynes (1937); for further rules of behaviour derived from Keynes see Falkinger (1983).
- 8 The degree of flexibility (costs of revision of decision) in the present model is exogenous; one might approach the Post-Keynesian design even more closely by endogenizing decision on the degree of flexibility and internalizing it into the utility maximization.
- 9 With this reciprocal fertilization of the Keynesian position and Neumann-Morgenstern Utility-Maximization we take a stance similar to Nermuth's (Nermuth 1983, p. 4): 'Effects ascribed to genuine uncertainty, incapable of being analysed according to Keynes, may just as well – if not better – be derived by standard theory.' Contrary to Lucas (1977), who considers the concept of rational expectations under Knight's uncertainty inapplicable: 'In cases of uncertainty, economic reasoning will be of no value.' The 'ingestion' of the Post-Keynesian position by the apparatus of the Neumann-Morgensternian utility-maximization does not deprive the Post-Keynesians of all merit and function. Were it not for their perpetual criticisms, standard theory would be ever more seduced by the elegance and solubility of mathematical constructions instead of making verisimilitude the loadstar of its models, would not let 'imperfections' such as price rigidities, disequilibria and two-phase decision processes enter into its model constructions. See Solow (1984) for a Keynesian who supports a formally consistent way to promote Keynesian ideas.
- 10 Overviews on the problems of, and alternatives to, expected utility maximization are to be found in the proceedings of the biannual conferences on the 'Foundation of Risk and Utility Theory' (Stigum & Wenstop 1982, Daboni *et al.* 1986).

4 General Propositions on the Influence of Uncertainty on Optimal Decisions

Optimal decisions under uncertainty are calculated – as under certainty – by the maximization of an objective function with respect to the decision variable. We will label the optimal value of the decision variable under uncertainty as \hat{Y} , as compared with Y^* in case of certainty. We will develop four classes of models under which we can make unambiguous assessments of the influence of uncertainty on the optimal decision, namely whether the optimal value will be higher, the same as, or lower than under certainty. If a model fits into one of these classes, we can make a statement as to the qualitative influence of uncertainty without calculating the optimality conditions explicitly.

We assume a utility function (4.1) in which utility U depends on the variable Z (which can be understood as profits): Z itself depends on two variables X and Y , (which usually are quantity produced and priced). In the world of certainty X_0 is known and there exists an optimal solution Y^* for the decision variable Y , under which profits are maximized (the second-order condition is assumed to hold).

$$\text{Max } U [Z(X_0, Y)] \rightarrow Y^* \text{ (certainty maximum).} \quad (4.1)$$

Under certainty we assume maximization of expected utility (Neumann–Morgenstern Utility-Maximization). Uncertainty exists about the variable X for which a probability density function $f(X)$ is known, its expected value is assumed to be the same as the fixed value X_0 under certainty (mean preserving introduction of risk).

$$\text{Max } E U [Z(X, Y)] \rightarrow \hat{Y} \quad \text{(uncertainty maximum).} \quad (4.2)$$

The optimal value of the decision variable labelled \hat{Y} under uncertainty can be shown to be smaller (equal, larger) than the optimal value under certainty Y^* , if U_{YXX} is smaller (equal or larger) than zero. (Rothschild & Stiglitz 1971, Diamond & Stiglitz 1974, etc.).

$$\hat{Y} \cong Y^* \quad \text{if } U_{YXX} \cong 0. \quad (4.3)$$

Unfortunately this condition is not very useful, since U_{YXX} proves for most problems to be neither unambiguously positive nor negative. It changes its sign in the domain of X for nearly every sensible problem (see Hey 1981 or Kraus 1979).

Under each of two modifications (operationalization A and B) unambiguous results are available. The predominant way in the literature is to assume a linear technology, i.e. $Z_{XX} = 0$.¹ This means that though the realization of the random variable X has an influence on Z , the expected value of Z does not depend on the degree of uncertainty.

Under the additional condition that the decision variable under certainty depends positively on the uncertainty variable, $dY^*/dX > 0$, we get the result of equation 4.4, that the optimal decision under uncertainty will be lower, (the same, higher) than under uncertainty, if people are risk-averse (neutral, loving).

Operationalization A: $Z_{XX} = 0$, $dY^*/dX > 0$:

$$U_{ZZ} \cong 0 \Rightarrow \hat{Y} \cong Y^*. \quad (4.4)$$

Proposition 1: Given that $dY^*/dX > 0$, and a linear technology ($Z_{XX} = 0$), then risk aversion (neutrality, loving) implies a lower (the same, a higher) optimal value for decision variable under uncertainty.

This result is not only known in literature, it is actually dominating articles on economic behaviour under uncertainty to a degree that it is often forgotten that there may be channels through which uncertainty influences behaviour other than risk attitude, and that the impact of risk attitude depends crucially on the sign of dY^*/dX . The condition $dY^*/dX > 0$ may be considered an innocent assumption if output and output prices are the relevant variables, but it cannot be considered as a matter of fact; see for example the relation between output volume and input prices.

The alternative way to get unambiguous results is to assume risk neutrality. The qualitative impact of uncertainty now depends on the sign of Z_{YXX} . If Z_{YXX} is positive, zero or negative, then the optimal decision under uncertainty is as given in equation 4.6.

The proof makes use of the assumption that the second-order condition for maximization under certainty holds ($Z_{YXX} < 0$) and of Jensen's inequality for convex or concave functions. We prove the case of $\hat{Y} > Y^*$ defining Z_Y concave in X .²

$$Z_Y(X_0, \hat{Y}) > EZ_Y(X, \hat{Y}) = 0. \quad (4.5)$$

The inequality holds for any concave function (and so for Z_Y), the equality stems from the first-order maximization under uncertainty.

It follows that $Z_Y(X_0, \hat{Y})$ is positive, and using $Z_{YY} < 0$ this implies that \hat{Y} is smaller than Y^* (where $Z_Y = 0$).

The result may also be derived from the Rothschild–Stiglitz condition, but though the derivation is mathematically trivial, it has to my knowledge never been done explicitly.

Operationalization B: $U_{ZZ} = 0$
 $Z_{YY} < 0$ (second-order condition under certainty)

Proposition 2: a linear utility function ($U_{ZZ} = 0$) and technological concavity, neutrality, convexity ($Z_{YXX} < 0, Z_{YXX} = 0, Z_{YXX} > 0$) yield the following sufficient condition:

$$Z_{YXX} \leq 0 \rightarrow \hat{Y} \leq Y^*. \quad (4.6)$$

The effect of uncertainty does not now depend on the risk attitude, but on elements of $Z(X, Y)$, these may be demand conditions, cost conditions or other elements of the production technology. We want therefore to label this channel as *technological concavity* (linearity, convexity) to stress the difference in attitude to that in proposition 1. Though other influences than risk attitude are not unknown in the literature, the trivial modification of the Rothschild–Stiglitz condition (4.3) to arrive at equation (4.6) has not been performed in the literature and the scattered models showing the relevance of uncertainty in the absence of risk aversion have up to now not been classified according to Z_{YXX} .

The attractiveness of the two operationalizations (A and B) lie in the fact that we can determine the qualitative influence of uncertainty without maximizing expected utility at all. It suffices to calculate Z_{XX} respectively Z_{YXX} for the certainty model.

Unfortunately both operationalizations cannot be used for disequilibrium models.³ We want to refer to disequilibrium models as models in which production and sales (demand) may differ. Competition with demand uncertainty, monopoly with *ex ante* control of prices and quantity and many inventory models belong in this category. In these cases production and demand differ, thereby creating a cost component under uncertainty which does not exist under certainty. Or in terms of the two operationalizations dY^*/dX respectively Z_{YXX} cannot be calculated since production (Y) and demand (X) are identical under certainty.⁴

We derive a fairly general result for this type of disequilibrium model below. We assume a certainty model (equation 4.7) in which revenue and costs depend on production (the components are separable), and derive the well-known first- and second-order conditions (equations 4.8 and 4.9).

Then we assume an uncertainty model in which expected sales depend on the smaller of demand (X) or production (Y) and we get optimality conditions 4.11 and 4.12.

The first-order condition, 4.11, has one additional element as compared to the certainty condition, 4.8, and this component is unambiguously negative. Therefore we get the very strong result, that optimal production will be lower under uncertainty as compared to certainty (4.13).

Certainty model:

$$\pi = r(Y) - c(Y). \quad (4.7)$$

$$\pi_Y = r'(Y) - c'(Y) = 0. \quad (4.8)$$

$$\pi_{YY} = r''(Y) - c''(Y) < 0. \quad (4.9)$$

Uncertainty model:

$$E\pi = \min[r(X), r(Y)] - c(Y). \quad (4.10)$$

$$\frac{\partial E\pi}{\partial Y} = \underbrace{r'(Y)}_{\substack{\text{marginal} \\ \text{revenue} \\ \text{under} \\ \text{certainty}}} - \underbrace{F(Y) \cdot r'(Y)}_{\substack{\text{marginal costs} \\ \text{of uncertainty}}} - \underbrace{c'(Y)}_{\substack{\text{marginal} \\ \text{costs} \\ \text{under} \\ \text{certainty}}} = 0. \quad (4.11)$$

$$\frac{\partial^2 E\pi}{\partial Y^2} = r''(Y)[1 - F(Y)] - r'(Y) \cdot f(Y) - c''(Y) < 0. \quad (4.12)$$

Proposition 3: Given a certainty model of the type 4.7 and an uncertainty disequilibrium model of the type 4.10, uncertainty adds an additional marginal-cost component which is positive (since the distribution function $F(Y)$ as well as $r'(Y)$ are positive), this yields, for this type of model, the unambiguous result of equation 4.13 (recall that $r''(Y)$ is smaller than $c''(Y)$ in the neighbourhood of Y^* by equation 4.9).

$$\hat{Y} < Y^*. \quad (4.13)$$

The economic rationale for this strong result is that there is an additional cost component under uncertainty, namely potentially unsold production or potentially unsatisfied demand. It is the very existence of this component, and not whether these extra components increase or decrease with the production chosen, which matters. This additional cost added to the usual upward-sloping, marginal-cost curve gives a crossing with the marginal-

revenue curve left of the certainty optimum. We will label the new component 'marginal costs of uncertainty'.

Proposition 3 allows the derivation of qualitative results for the competitive model with demand uncertainty (see section 5.2.4), and also for models where demand is negatively related to price or for some cases of uncertainty about productivity (5.4). Components similar to 'marginal costs of uncertainty' are present in the $p-q$ *ex ante* mode for the monopolist and in most inventory models. In all these models the very presence of uncertainty leads to a factor tending to reduce optimal production due to the potential disequilibria. No third derivative is necessary as in proposition 2.

One characteristic of the model has to be mentioned, which may limit its use and which may explain why it has not been investigated more thoroughly in the literature. The exogenous price in both the certainty and in the uncertainty model have to be identical to allow comparisons. This seems to be a reasonable assumption for partial models – such as are used in this book – but may not be the case in more general models.

The fourth factor that could change the optimal production is given if it is possible to make a *preliminary* decision about the decision variable; and then, after that veil of uncertainty is lifted, to revise this decision at some cost. It is easy to understand that if the cost of revising the decision upwards is larger than that of downward revision the preliminary optimal production will rise, in the other case it will fall. Downward irreversibility of gross investment is one related form of asymmetry.

Proposition 4: Suppose it is possible to make a preliminary decision \hat{Y} and revise this upward (downward) at cost c_1 (c_2) then $c_1 \leq c_2$ tends to imply $\hat{Y} \leq Y^*$.

This source of asymmetry is unattractive from the theoretical point of view: it seems to be almost too trivial. Nevertheless, no general model is available which models the effect of asymmetric *ex post* flexibility in a general way, so we had to use the words 'tend to' in proposition 4. In chapter 8 we develop a linear model in which asymmetric *ex post* adjustments of the optimal are feasible. For non-linear equilibria models the result depends (as shown by Turnovsky 1973) on the interaction of 'normal production costs' and 'emergency cost' (third cross derivative). (For irreversibility of downward investment see section 7.3.2.)

From the practical point of view upward revisions seem often to be much easier (less costly) than downward revisions. Reselling production, getting rid of investment goods, laying off personnel in the short run (especially in business troughs or facing shocks affecting a whole industry), usually proves very difficult.

Let us sum up. We have found four propositions under which general rules forecasts on the impact of uncertainty are available. Risk aversion, technological concavity, disequilibria and asymmetric flexibility with high losses

in case of too high values of the decision variable will tend to bias down the optimal value of the decision variable. Risk-loving, technological convexity and higher losses in case of too low values of the decision variable will tend to increase the optimal value under uncertainty.

Three of the four propositions refer to 'objective' factors related to costs, demand curves or other model characteristics, but not to risk attitude. We will call these factors 'technological reasons in the wider sense' to stress that it is not the subjective attitude towards risk (about which the economist does not have much information and even less right to make normative assumptions) which decide in these models. Stressing the objective factors does not mean that we deny that people may be risk-averse in many cases (see section 12.3 for very rough empirical evidence) on risk attitude. There are, however, reasons for this emphasis since

- the literature heavily concentrates on the risk attitude;
- there is little *a priori* information on risk attitudes, and these may change according to wealth and/or the riskiness of a decision; and
- under repeated decisions or choosing among diversified assets, firms should behave as if they were risk-neutral (see for example Nickell 1978, p. 8).⁵

The technological factors on the other hand are independent of subjective attitudes and relevant under repeated decisions. This may be of special importance if we use the results of these models to explain downward-biased expectation. Expected sales biased downward due to risk aversion could be dismissed into the group of errors due to psychological reasons in a sense, expected sales biased downward due to cost, demand and market structure are the greater challenge for the economists as a profession.

NOTES

- 1 Let it be remembered that these concepts of 'linear' and 'non-linear' technology differ from those used in production theory (where concavity and convexity respectively of the production function are termed 'non-linear' technology).
- 2 The derivation of equation 4.6 results, for example, directly from the equation for U_{XXY} (quoted by Hey 1981, p. 44), but there a follow-up of this special case, where U is linear in Z , is rejected for being trivial. The one-sided interest of economics in the consequences of risk attitude also becomes apparent when Lippmann & McCall (1981, p. 212) in an article on the state of uncertainty theory feel obliged to point out that other factors besides risk attitude can make uncertainty theory interesting: '... though in many circumstances risk aversion is a fact . . . , much economic behaviour is a direct consequence of uncertainty and is independent of risk aversion.'
- 3 I am grateful to Manfred Nermuth from the University of Bielefeld for the statement that a 'failure' of operationalization must not be understood to mean that uncertainty in this case is not accessible to interpretation.

- 4 The case that price will drop so low that demand is deliberately not produced, is usually excluded in the literature (cf. Leland 1972, Karlin *et al.* 1962, Hey 1979).
- 5 See, for example, Nickell (1978, p. 84): 'If there exist firms with identically but independently distributed random profits, then as $n \rightarrow \infty$ any risk-averse investor will wish each and every firm to maximize its expected profit.' The existence of large firms or the correlation of profits over a business cycle restricts application of this rule in practice.

5 The Theory of the Firm under Uncertainty (One- period Models for Optimal Output or Price Decisions)

5.1 INTRODUCTION

Even within the framework of expected utility maximization, the number of ways in which the decision of firms can be modelled under uncertainty are abundant and so are the papers written on this subject. One can model different market structures (the 'structure') like monopoly, oligopoly, monopolistic competition or competition proper; we have to conjecture which variable has to be decided upon *ex ante* (the 'mode'); uncertainty can exist about output prices or demand, about productivity or input prices or the availability of one or more factors (the choice of the random variable). Some models contain a variable which can be adjusted (or adjusts automatically) after the veil of uncertainty is lifted to guarantee the identity of production and demand, some models allow disequilibria, some partial adjustments (the 'flexibility' issue). Uncertainty can be modelled in general functional forms, if specific forms are used, we have to decide whether uncertainty occurs in an additive way or in some multiplicative type (the 'type').

Since we can combine many – albeit not all for logical reasons – kinds of 'structures', 'modes', 'types', 'degrees of flexibilities' and random variables, we can imagine how many models can be constructed.

As far as the 'structure' is concerned we will restrict ourselves to the cases of competition and monopoly, though we think that the one of our competitive model with demand uncertainty may be (and in literature is) considered a fair representation of oligopolistic strategy and that the monopoly model can incorporate elements of monopolistic competition. As far as the choice of the decision variable – the 'mode' – is concerned, we concentrate on the output decision; in some cases we deal with output price or both price and output as the decision variables ($p-q$ -mode). Input choices are deferred to chapter 7. We will deal with disequilibrium models and partial flexibility in chapter 8. We cannot help dealing with the crucial question of the exact

specification of the uncertainty term – the ‘type’ issue – since the results are not only sensitive to the question whether the uncertainty variable is added in an additive or multiplicative way, but also because even for the multiplicative type alternative sub-types are available.

We restrict ourselves in this chapter to one-period models (inventory models will be dealt with in chapter 6, dynamic investment models in section 7.3). We will use comparable notations for all models and show that the results can be derived for the majority of the models from one of the four propositions derived in chapter 4. In the literature, the results were arrived at in various and imaginative ways, despite the fact that most papers were written after the Rothschild–Stiglitz condition had been published. We will try to find which empirical facts are consistent with the model and which would contradict its assumptions as well as its forecasts in order to assess the relevance of the models in later parts of the book. We repeat that we use a partial approach since we optimize the decision of one side of the market.

5.2 COMPETITION

5.2.1 Conceptual Problems of Competition under Uncertainty

The competition model under certainty assumes, as we know, that the enterprise accepts the (uniform) market price and at that price can sell any chosen quantity. Profit maximization is attained at that output volume where marginal costs are identical with the (known) market price.

The competition situation under uncertainty poses conceptual problems even if the question of uncertainty as to input decision (or about the production function) is disregarded. The simplest model, most frequently dealt with in the literature, assumes price uncertainty¹ and retains the assumption that any chosen output can be sold. In this case the optimal output (*Q ex ante* mode) that has to be decided upon *ex ante*. An *ex ante* setting of the price (in uncertainty about the market clearing price) is not compatible with optimizing behaviour: if the selected price lies below the subsequently established market price, possible earnings are forgone, if it lies above, nothing can be sold.

The assumption of quantity uncertainty – uncertainty about demand – appears at first sight to contradict the basic idea of a competition model, because this sets out from the marketability of any output; therefore some authors (e.g. Hey 1979, p. 133) reject demand uncertainty as ‘logically incompatible’.² Others (e.g. Hyman 1966) accept demand uncertainty in a competition model; most papers in inventory theory assume demand uncertainty under given prices without discussing the mechanism that makes the price exogenous.

The more recent disequilibrium theory describes models identical in structure and outcome with the competition model under demand uncertainty, as fix-price models under stochastic rationing (cf. Malinvaud 1980, p. 29ff, esp. formula 11, Benassy 1982, p. 45ff, and appendix C, Costrell 1983).

The rationale of price rigidity can be manifold. Of the authors who present the model of quantity uncertainty under competition, Hymans (1966) mentions the Cyert and March theory of the 'behavioural firm' that sees in price rigidity an attempt to reduce uncertainty. Hymans follows this up with his own concept of simultaneous price changes in industry overall, individual suppliers have no intention of increasing their market share and disposing of their eventual overproduction. The simultaneous price change of the whole industry is not quick enough to prevent an oversupply of goods making market clearance at short notice impossible. The short-term price rigidity in a competitive market may also stem from institutional causes (price regulation) or from the high cost of changing price (cf. the summary article by Amihud & Mendelson 1983, or the discussion on rational price rigidities in chapter 12 of this study).

Price rigidity in the oligopoly model as a possible mode of behaviour (beside price war) is well documented, it may – even without agreement – be an optimal strategy to preserve attained equilibrium (cf. for example, Scherer 1970, p. 25, where price rigidity is justified on grounds of the fragility of coordination problems and the kinky demand curve). Given rational causes for at least the short-term fixing of prices and market shares in oligopoly, the so-called competition model under demand uncertainty may capture some features of oligopoly behaviour under uncertainty.

However, price rigidities need not be caused by monopolistic or oligopolistic elements, inertia or information costs; they may be reactions to the uncertainty situation. Rothschild (1981, p. 114) says: '. . . firms' price setting behaviour does not – under uncertainty – conform to the "rules" of the equilibrium model, and that for good "rational" reasons. Prices are not as quickly as possible adapted to the prevailing state of the (positive or negative) surplus demand but are kept arrested over a longer period. This facilitates buyers' and sellers' planning for the shorter-term (uncertain) future. . . .' Further reasons for such behaviour are given (cf. Rothschild 1981 or Okun 1980).

The model of section 5.2.4 shall be called for short the 'competition model under demand uncertainty', but its propositions are valid for price rigidity howsoever caused, be it one that can be tolerated in a competition model, that it results from oligopoly behaviour, be it information or transaction cost caused, enforced by price regulation or an optimal reaction to reduce uncertainty.

In the terminology of disequilibrium theory it would be designated a *fix-price model with stochastic rationing*.

In the comparison here undertaken between certainty models and uncertainty models the respective exogenous price p is taken to be identical in both situations. In the partial analysis here presented (more exactly, a market side's partial analysis) this is probably the most obvious assumption. It would seem most realistic for the initial phase of an uncertainty situation (when the old prices are still valid). Generally speaking, in a more general uncertainty model the equilibrium price in the certainty model need not correspond to the equilibrium price under uncertainty.

5.2.2 Competition, Price Uncertainty, q -mode

Sandmo (1971) may be quoted as the classical reference for this model. Contrary to him and in keeping with the structure of our study we shall initially assume risk neutrality.

The producer faces an uncertain output price, but has some notion about the probability distribution of possible prices: $f(p)$. He must determine his output before the actual price is known, and will then accept the market price. He is a profit maximizer (equation 5.1) and chooses his quantity in such a way that marginal costs correspond to the expected value of the price (equation 5.2):

$$E\pi = \int_0^{\infty} [pq - c(q)] f(p) dp, \quad (5.1)$$

$$\frac{\partial E\pi}{\partial q} = \int_0^{\infty} [p - c'(q)] f(p) dp = Ep - c'(q) = 0, \quad (5.2)$$

where π is profit, p is unit price, $c(q)$ is cost function, and q is output (here also demand $x=q$).

Assuming that the expected price equals actual price under certainty, $p_0 = E(p)$, then produced quantity is equal under uncertainty and certainty ($\hat{q} = q^*$).

The result may also be derived from the calculation of Z_{YXX} . The maximization of the objective function under certainty (5.3) with respect to $Y(Y=q)$ results in the well-known marginal condition (5.4). The two-fold differentiation by $X(X=p)$ shows the technology of this model to be linear.³

$$Z = \pi = p \cdot q - c(q) \text{ (objective function under certainty),} \quad (5.3)$$

$$Z_Y = \pi_q = p - c'(q), \quad (5.4)$$

$$Z_{YXX} = \pi_{qpp} = 0 \Rightarrow \hat{q} = q^* \text{ (according to proposition 2 in chapter 4).} \quad (5.5)$$

The economic substantiation of the outcome lies in the fact that a contingent higher price would increase the profit to the same extent that

a lower price would decrease it, consequently expected profits are equal to actual profits under certainty. Expected profits cannot be increased by higher or lower output, therefore action is the same as under certainty.

5.2.3 Non-linear Objective Function

Under non-linear objective function (risk-aversion, risk-loving) maximization of expected utility will result in an outcome that differs from maximization of expected profit. This was demonstrated by Sandmo for risk aversion ($U'(\pi) > 0, U''(\pi) < 0$), and, somewhat modified by Hey (1979, p. 128f), the result is 'mirror-inverted' for risk-loving (equation 5.12ff).

The following equations show the objective function (5.6) and its first (5.7) and second (5.8) derivative. Since total output can be sold, output and quantity sold are identical ($q = s$; where $s =$ quantity sold).

$$EU(\pi) = \int_0^{\infty} U[pq - c(q)] f(p) dp = EU [pq - c(q)], \quad (5.6)$$

$$\frac{\partial EU(\pi)}{\partial q} = E\{U'(\pi) \cdot [p - c'(q)]\} = 0, \quad (5.7)$$

$$\frac{\partial^2 EU(\pi)}{\partial q^2} = E\{U''(\pi) \cdot [p - c'(q)]^2 - U'(\pi)c''(q)\} < 0. \quad (5.8)$$

Next, the term with $c'(q)$ in equation 5.7 is brought to the right, $E[U'(\pi) \cdot Ep]$ is deducted:

$$EU'(\pi) (p - Ep) = E\{U'(\pi) [c'(q)] - Ep\}. \quad (5.9)$$

From equation 5.10 follows condition 5.11:

$$\pi - E\pi = (p - Ep)q, \quad (5.10)$$

$$\pi \cong E\pi \text{ if } p \cong Ep. \quad (5.11)$$

From the assumption of risk aversion the following equation (5.12) results (the opposite is true under $U''(\pi) > 0$):

$$U'(\pi) \cong U'(E\pi) \text{ if } p \cong Ep. \quad (5.12)$$

From this follows the weak inequality (5.13) and for the expected values follows the strong inequality (5.14):

$$U'(\pi) (p - Ep) \leq U'(E\pi) (p - Ep), \quad (5.13)$$

$$E[U'(\pi) (p - Ep)] < U'(E\pi) \underbrace{E(-Ep)}_0. \quad (5.14)$$

The right side of equation 5.14 is zero, the left side of 5.14 is also the left side of equation 5.9; because of the equality in 5.9 this requires that the right side of 5.9 must also be smaller than zero. This happens only if $c'(q) < Ep$.

$$c'(q) < Ep. \quad (5.15)$$

Given rising unit costs (convex costs) the competitive firm under price uncertainty will produce, under risk aversion (risk-loving), less (more) than under certainty. (For non-convex costs the output under certainty is not defined.) This ascertained tendency is recorded in equations 5.16 and 5.17:

$$\hat{q} < q^* \quad \text{if } U''(\pi) < 0, c''(q) > 0, \quad (5.16)$$

$$\hat{q} > q^* \quad \text{if } U''(\pi) > 0, c''(q) > 0. \quad (5.17)$$

The economic interpretation of the differing outcomes, depending on the risk attitude, is the following: once the decision on the output quantity to be produced has been taken, production costs are fixed while profit fluctuates solely because of the differential revenues per unit sold. Through the transition from the known price under certainty to the price range under uncertainty the expected profits remain the same, but their dispersion increases. Risk neutrality means, roughly, indifference *vis-à-vis* greater dispersion, therefore the same quantity can be produced as under certainty. (Profit maximization is effected at the same output as maximization of expected profit.)

Risk aversion attaches co-decisive influence (aversion) to greater variance; maximum utility is attained at lower output. Mathematically, the result derives from the concavity of the objective function.

The result of the shift of optimal output as against certainty (which is shown under strictly convex/concave utility function) must be modified, if there are forward markets. Holthausen (1979) shows that – given the possibility of selling at least part of the output in forward markets at a known price (b) – the output of a firm acting under price uncertainty (even given the firm's risk aversion), will approach output volume under certainty. The output (calculated from $c'(q) = b$) depends solely on future price; if the latter is near the expected value of the current price (Ep), then price uncertainty in conjunction with risk aversion does not influence output.

5.2.4 Competition, Demand Uncertainty, q -mode (= Fix-price Model with Stochastic Rationing)

We have already referred (in section 5.2.1) to the conceptual difficulties of this model, but also to its possible realism. Here we want to present first the case of risk neutrality and then the case of non-linear utility function. In all models we must distinguish between sold quantity (quantity

demanded x), and produced quantity q , since – due to price rigidity – output need not necessarily be completely sold. As we are dealing with a one-period model, it is implicitly or explicitly imputed that the ‘goods left over’ have no value (are not storable), and for the same reason, that no value is ascribed to unsatisfied demand (demand cannot be backlogged).

The profit maximizer’s expected profit results from the difference between costs and expected revenue where revenue is the product of price and quantity demanded, or produced (whichever is the smaller, see equation 5.18).

$$E\pi = p \cdot \min[q, x] - c(q) = \int_0^q p \cdot xf(x)dx + \int_q^\infty p \cdot qf(x)dx - c(q). \quad (5.18)$$

By differentiation with respect to output we arrive at the marginal condition (5.19). This latter is subject to the following formally identical interpretations (equations 5.21a–d):

$$\frac{\partial E\pi}{\partial q} = \int_q^\infty pf(x)dx - c'(q) = 0. \quad (5.19)$$

$$\frac{\partial^2 E\pi}{\partial q^2} < 0. \quad (5.20)$$

$$p [1 - F(q)] = c'(q). \quad (5.21a)$$

$$[1 - F(q)] = \frac{c'(q)}{p}. \quad (5.21b)$$

$$F(q) = \frac{p - c'(q)}{p}. \quad (5.21c)$$

$$p = \underbrace{c'(q) + pF(q)}_{\text{marginal cost of uncertainty}}. \quad (5.21d)$$

$$\hat{q} < q^* \quad (\text{if } c''(q) > 0), \quad (5.22)$$

- the expected marginal revenue is equated to marginal cost, expected marginal revenue being defined as marginal revenue (= price) multiplied by the probability that the last produced product can also be sold (5.21a); remember $F(q)$ = distribution function;
- the probability that the last unit will be needed ($1 - F(q)$), should equal the ratio of marginal cost to price (5.21b);
- the probability of overproduction, $F(q)$, should equal the quotient of marginal profit [$p - c'(q)$] to price (5.21c);
- price is equated with an extended concept of marginal cost in that, beside the marginal cost of production, the probability that q cannot be sold (‘marginal cost of uncertainty’) is also taken into consideration (5.21d).

The cause for the downward shift of output is that anticipated marginal revenue under uncertainty is not only equated with marginal production costs, but with marginal production costs plus a cost component resulting from the probability of the non-marketability of the production. This model is a special case of the model used in operationalization 3 in chapter 4, where $r'(y) = p$.

Price is exogenous, and to ensure comparability is assumed to be equal in the certainty and in the uncertainty model. For the uncertainty model here under consideration it is the most plausible assumption: it probably gives an adequate representation of entrepreneurial behaviour given uncertainty (the 'prevailing' price is maintained, at least temporarily). But we want to repeat that, generally, the equilibrium price in the uncertainty model need not necessarily equal the equilibrium price in the equilibrium model.

5.2.5 Non-linear Utility Function

For non-linear utility functions, quantity uncertainty and q -mode we follow the presentation of Hymans (1966); again one must distinguish between quantity produced (q) and quantity demanded (x). The objective function (maximization of expected utility) is expressed in equation 5.23; by differentiation with respect to (q) and supplementing both sides by

$$\int_0^q U' [px - c(q)] f(x) dx, \text{ equation 5.25 will result:}$$

$$EU = \int_0^q U [px - c(q)] f(x) dx + \int_q^\infty U [pq - c(q)] f(x) dx. \quad (5.23)$$

$$\frac{\partial EU}{\partial q} = \int_0^q U' [px - c(q)] f(x) dx [-c'(q)] + \int_q^\infty U' [pq - c(q)] f(x) dx [p - c'(q)] = 0. \quad (5.24)$$

By trivial recasting this equation can be made comparable in form to equation 4.3 in the preceding chapter. Now the ratio of marginal profit to price will be equated, not with $F(q)$, but with $L(p, q)$.

$$\frac{[p - c'(q)] \int_0^q U' [px - c(q)] f(x) dx}{p - c'(q)} + \frac{\int_0^q U' [px - c(q)] f(x) dx \cdot c'(q)}{p - c'(q)} =$$

$$\frac{\int_0^q U' [pq - c(q)] f(x) dx + \int_0^q U' [px - c(q)] f(x) dx}{EU' [pS - c'(q)]} \quad (5.25)$$

$$\frac{p - c'(q)}{p} = \frac{\int_0^q U' [px - c(q)] f(x) dx}{EU' [pS - c'(q)]} = L(p, q), \quad (5.26)$$

where S is sales (volume), and $ES = \int_0^q xf(x)dx + q \int_q^\infty f(x)dx$ is expected sales.

The function $L(p, q)$ represents the relation of marginal utility under low demand (cumulated between 0 and q) to expected value of marginal utility from expected revenue (wherefore the denominator is made up of the same term as the numerator plus marginal utility of a produced unit under high demand – cumulated above q to infinity).

For $q=0$ the function, too, is 0; it approaches 1 when q approaches infinity. It follows that optimal output (if $c'(q)$ does not equal zero) is always finite, and lower than output under certainty (where $p = c'(q)$). This outcome holds true under concave, linear and convex utility function. The following hierarchy exists between output under risk aversion, neutrality, loving and under certainty:

$$\hat{q}_{RA} < \hat{q}_{RN} < \hat{q}_{RL} < q^*; \quad (5.27)$$

where RA, RN, RL represent strict risk aversion, risk neutrality and strict risk loving respectively.

The economic argument for the ranking of the first three choices is this: the risk-averse businessman reduces output in order also to reduce the variance of the eventual outcomes; the risk lover increases output because by so doing his chances will also rise. Reduction in all these three alternatives as against the case of certainty takes place because in the case of uncertainty an additional cost component exists. 'Marginal costs of uncertainty' are positive and proposition 3 determines the outcome.

5.3 MONOPOLY

5.3.1 Conceptual Problems in Monopoly under Uncertainty

The monopolist (and equally the supplier in monopolist competition – cf. Lim 1980, p. 217) faces a negatively sloped demand curve. Uncertainty consists in the demand curve not being known for certain, in other words, that dependence of price on quantity (and vice versa) is only (conditionally) given for every state of nature. The question whether price uncertainty or quantity uncertainty prevails no longer presents itself. However, a new question arises, namely in what manner the relation between quantity and price is overshadowed by uncertainty (the 'type' issue).

Two possibilities present themselves where the second has to be subdivided again):

- uncertainty is additively superimposed over the demand curve $x = f(p) + u$; or
- uncertainty is multiplicatively superimposed over the demand curve type A: $p = g(x) \cdot u$; or type B: $x = f(p) \cdot u$.

Both forms of uncertainty (additive and multiplicative respectively) will in certain circumstances give different outcomes for the behaviour comparison under certainty and uncertainty.⁴ This outcome is familiar in the literature (cf. Karlin & Carr 1962, Zabel 1972). The fact, however, that the two different specifications of multiplicative uncertainty will also cause different outcomes has not been pointed out. (On the possibility of different outcomes for types A and B respectively, see sections 5.3.2.3 and 5.3.2.4.)

The second conceptual problem is the question of choice of the action variable (the mode issue).

In monopoly under certainty both a quantity and a price-setting strategy is possible, but both strategies lead to the same decision and the same profit. Under uncertainty this is no longer the case. We therefore distinguish three modes of behaviour:

- the monopolist sets output before demand is known (*q ex ante* mode) and thereafter lets the (actual) demand curve decide the market clearing price;
- the monopolist sets price before demand is known and thereafter lets the (actual) demand curve decide on market-clearing output (*p ex ante* mode);
- the monopolist fixes price and quantity before demand is known (*p, q ex ante* mode), output can be sold or not ($q \geq x$), and demand is fully satisfied or not ($q \leq x$). Disequilibria may therefore exist.

An additional decision faces the monopolist: whether he shall produce *any* demanded quantity (under *p*-mode) and /or if he wants to sell any quantity produced (under *q*-mode). Most studies assume (and so do we) that goods demanded are produced in all cases and that goods already produced are sold in any case.

Production of any demanded quantity under the *p*-mode offends against profit maximization if the price lies below marginal costs.⁵

The second problem presents hardly any difficulty (as long as price is positive) in one-period models. But in the multi-period model that assumption is very problematical, since storing is a possibly profit-increasing alternative.

The choice of the mode *per se* may either depend on external circumstances (obligation or commitment to supply, price regulation, etc.), or it must, itself, also be subject to microeconomic maximization. Profits and utility may be different under all strategies. Lim stated that the *q*-mode is definitely preferable to the *q-p*-mode, but the *p*-mode not under all circumstances (in view of the danger of high marginal production costs possibly exceeding the yield from greater flexibility, Lim 1980).

The expected profitability of a *p*- to a *q*-mode is determined by the cost curve (convex costs will make the *q*-mode more attractive).

5.3.2 Monopoly, *q*-mode, Price Taker

In these models the monopolist determines output (*q*-mode) and (after the veil of uncertainty has lifted) lets demand determine the market-clearing

price. That decision taken, costs are fixed and revenue alone fluctuates with the state of the market.

Since the letter f stands for the demand function, we shall deviate in this section from the rest of the study and shall use the Greek letter ρ to express the density function.

5.3.2.1 Leland (1972)

Standard reference for this model is Leland (1972). Leland arrives at his findings with two qualifications of the universality of the problem formulation. The general implicit demand function would be $f(p, q, u) = 0$, Leland assumes that the higher values of uncertainty u are connected with higher demand. This implies the following conditions:

$$\text{when } p = p(q, u) \text{ then } \frac{\partial p(q, u)}{\partial u} > 0. \quad (5.28)$$

$$\text{when } q = q(p, u) \text{ then } \frac{\partial q(p, u)}{\partial u} > 0. \quad (5.29)$$

Next, Leland introduces the principle of increasing uncertainty (PIU): 'as total revenue increases (for changes in p or q), it seems natural to expect that the "riskiness" or dispersion of total revenue will increase.'

He shows that PIU is tantamount to the condition that for all u the algebraic sign of marginal yield (with respect to u) equals the algebraic sign of expected marginal yield.

Under these conditions Leland comes to the constantly quoted result that – for the quantity-setting monopolist – quantity produced under uncertainty and under certainty are equal.

$$\hat{q} = q^* \text{ given the assumptions: 5.28, 5.29, PIU.} \quad (5.30)$$

Neither Leland nor his followers in the literature are fully aware of the restrictive character of the *assumptions*, e.g. of the fact that the outcomes are not valid for the multiplicative case of type B. In his footnote 4 (Leland 1972, p. 279), he suggests that the premises (especially 5.28 and 5.29) are fulfilled for all forms of multiplicative demand and that the PIU, 'although not satisfied in all instances, has strong intuitive appeal' (ibid.).

5.3.2.2 Additive Uncertainty

For additive demand Leland's finding may be demonstrated through maximization of expected profit (equations 5.31–5.33) or application of the operationalization 2 (Z_{YXX} , see equations 5.34 and 5.35).

Equation 5.31 defines additive uncertainty, equation 5.32 the expected profit that is to be maximized. Since this corresponds to the profit to be

maximized under certainty, the decision remains unaffected by the given form of uncertainty (cf. equation 5.33, but also application of operationalization 2 in equations 5.34 and 5.35).

$$p = g(q) + u_1 \quad E(u) = 0. \quad (5.31)$$

$$E\pi = \int_{-\infty}^{\infty} \rho(u) \{ [g(q) + u]q - c(q) \} du, \\ \int_{-\infty}^{\infty} \rho(u) u du = E(u) = 0, \\ \int_{-\infty}^{\infty} \rho(u) du = 1, \rho(u): \text{density function.} \quad (5.32)$$

$$E\pi = \pi^{\text{cert}} = g(q) \cdot q - c(q). \quad (5.33)$$

$$Z_Y = \pi_q = g(q) + g'(q) \cdot q - c'(q). \quad (5.34)$$

$$Z_{YXX} = \pi_{quu} = 0 \Rightarrow \hat{q} = q^*. \quad (5.35)$$

5.3.2.3 *Multiplicative Uncertainty, Type: $p = g(q) \cdot u$*

Given multiplicative demand of this type, when price is a function of quantity times an uncertainty term (with an expected value 1, see equation 5.36), uncertainty can change either costs or expected revenue (cf. equation 5.37), and the decision is identical with decision under certainty (equation 5.38).

The outcome follows from proposition 2 (see equation 5.39):

$$p = g(q) \cdot u, \quad E(u) = 1, \quad (0 \leq u \leq 2). \quad (5.36)$$

$$E\pi = \int_0^2 \{ \rho(u) [g(q) \cdot u] q - c(q) \} du, \quad \int_0^2 \rho(u) \cdot u du = Eu = 1. \quad (5.37)$$

$$E\pi = \pi^{\text{cert}} = g(q) \cdot q - c(q). \quad (5.38)$$

$$Z_{YXX} = \pi_{quu} = 0. \quad (5.39)$$

5.3.2.4 *Multiplicative Uncertainty, Type: $q = f(p) \cdot u$*

A minimal change in the specification of uncertainty changes the outcome. If the demand curve under uncertainty is specified in the form of type B, this will result in a different implied price equation than in the preceding subsection (cf. equation 5.31 and 5.36). This formulation formally runs counter to Leland's assumptions (at least for an essential range of demand functions), though to my mind there is no *a priori* plausibility that speaks in favour of one or other type.⁶ Nickell (1978), for example, without discussing economic rationality, chooses the type described in this subsection, nor does he emphasize his outcome's difference from that of Leland (1972).

With this type of multiplicative uncertainty expected costs remain unaffected by uncertainty (since they were determined *ex ante* in the

decision), but expected revenue is influenced by the form of the demand curve. If marginal revenue is concave in u , expected marginal revenue $EMR(q, u)$ declines. In order to keep it even with marginal costs, output must be reduced to below certainty quantity.

The specific type of uncertainty is described in equation 5.40, the function to be maximized is represented in equation 5.41, the result of the maximization in equation 5.42. We assume $f(\cdot)$ to be invertible. For the shorter derivation of the result from the certainty model according to proposition 2 (operationalization 2) see equations 5.43 and 5.44.

$$q = f(p) \cdot u \Rightarrow p = f^{-1} \left(\frac{q}{u} \right), E(u) = 1. \quad (5.40)$$

$$E\pi = \int \varrho(u) \left[f^{-1} \left(\frac{q}{u} \right) q - c(q) \right] du. \quad (5.41)$$

$$\frac{\delta E\pi}{\delta q} = \int \varrho(u) \underbrace{\left[f^{-1} \left(\frac{q}{u} \right) + \frac{\partial f^{-1} \left(\frac{q}{u} \right)}{\partial q} q \right]}_{EMR(q, u)} du - c'(q). \quad (5.42)$$

$$Z_Y = Z_q = MR(q, u) - c'(q). \quad (5.43)$$

$$Z_{YXX} = Z_{quu} < 0, \quad \text{if } MR(q, u) \text{ concave in } u. \quad (5.44)$$

For instance, $MR(q, u)$ is then concave, when the demand function is linear (cf. equation 5.45a-c) or when it is quadratic, or when demand elasticity is constant and smaller than -1 (these are sufficient conditions; for necessary and sufficient conditions see Nickell 1978, p. 91).

Sufficient conditions for $MR(q, u)$ concave in u :

$$f(p) \text{ linear, or} \quad (5.45a)$$

$$f(p) \text{ quadratic, or} \quad (5.45b)$$

$$\epsilon < -1. \quad (5.45c)$$

The statistical background of the outcome consists in the uncertainty term entering into the argument of the implicit demand function (as a quotient) instead of standing 'outside' the demand function. Consequently, the demand function's features become significant and create technological concavity. The economic significance of the specification of uncertainty in this form is that, for a given u , price variation increases with increasing quantity produced, the concavity of marginal revenues resulting in the advantages of higher prices being more than compensated for by the disadvantages of lower prices.

5.3.3 Monopoly, *p*-mode, Quantity Taker

In this model the monopolist sets the price in the course of maximizing expected profit and lets the demand curve decide on the quantity to be produced.

5.3.3.1 Leland (1972)

Again Leland is the standard reference, and he shows that

... the introduction of uncertainty does not affect the price decision of the price-setting, risk-neutral firm with constant marginal cost. (Leland 1972, p. 285)

... when marginal costs are not constant ... Jensen's inequality may be used to show that if marginal cost is rising at a non-decreasing rate, the optimal price set by risk-neutral firms will be higher under uncertainty than under certainty; the opposite holds if marginal cost is decreasing at a non-increasing rate ($c'' < 0; c''' \leq 0$). (ibid).

We shall demonstrate that Leland's statements, though much quoted (see Hey 1979, p. 136ff) are, again, valid only under additive and under specific types of multiplicative uncertainty, and, second, that the equals sign for c''' in the final quote is misleading in the additive case.

5.3.3.2 Additive Uncertainty

Under additive uncertainty optimal price depends on the cost function. If $c' [q(p, u)]$ is concave (linear, convex) in u , then optimal price is lower, equal, higher than under certainty.

$$q = f(p) + u. \quad (5.46)$$

$$E\pi = \int \varrho(u) [p \cdot (f(p) + u) - c(f(p) + u)] \quad (du = p \cdot f(p) - \int c[f(p) + u] \varrho(u) du. \quad (5.47)$$

In view of the symmetry of $\varrho(u)$, the difference between profit maximization and maximization of expected profit can be presented in the form of equation 5.48. The third derivative of the cost curve decides on the chosen price under uncertainty as relative to optimal price under certainty (5.49). This may also be shown through equations 5.50–5.52.

$$\frac{\partial \pi^{\text{cert}}(p)}{\partial p} - \frac{\partial E\pi(p)}{\partial p} = \int \varrho(u) \left\{ \frac{1}{2} c' [f(p) + u] + \frac{1}{2} c' [f(p) - u] - c' [f(p)] \right\} f'(p) du. \quad (5.48)$$

$f'(p) < 0$, assumption of negatively sloped demand curve under uncertainty.

$$c''' \geq 0 \Rightarrow \hat{p} \geq p^* \tag{5.49}$$

$$Z = p[f(p) + u] - c[f(p) + u] \tag{5.50}$$

$$Z_p = f(p) + u + pf'(p) - c'[f(p) + u] \cdot f'(p) \tag{5.51}$$

$$Z_{pu} = 1 - f'(p) \cdot c'' [f(p) + u] \tag{5.52}$$

$$Z_{puu} = \underbrace{-f'(p)c'''}_{+} [f(p) + u] \tag{5.53}$$

$$Z_{puu} \geq 0 \text{ if } c' \left\{ \begin{array}{l} \text{convex} \\ \text{linear} \\ \text{concave} \end{array} \right\} \text{ in } u. \tag{5.54}$$

To determine the outcome in equation 5.48 the difference is used between curves Z_p and EZ_p respectively: when c' is convex in u , then the term in the curled bracket is positive, and since $f'(p)$ is negative, curve EZ_p must always lie higher than curve Z_p . It will therefore intersect the x axis later, and hence the price under uncertainty is higher than under certainty.

Equations 5.50–5.54 demonstrate the outcome by calculating Z_{YXX} . The third derivative must be strictly bigger or smaller than zero to guarantee a distortion ($c''' \leq 0$ does not suffice, as claimed by Leland 1972, p. 285, for the more general case).

The economic background to the shift between price under certainty and uncertainty lies in that marginal revenue (which is equal to marginal revenue under certainty) is to be equated to expected marginal costs. When a hypothetical quantity q is replaced by a batch of equally probable higher and lower outputs, expected marginal costs are equal to MC under certainty as long as the marginal cost curve is linear (be the total cost curve convex or concave). Not before the marginal-cost curve becomes convex do anticipated marginal costs rise on replacement of q_0 by $(q_0 + \epsilon)$ and $(q_0 - \epsilon)$. The higher price chosen is insurance to reduce the danger of having to produce in the range of high marginal costs if demand should turn out to be brisk.

5.3.3.3 Multiplicative Uncertainty, Type: $q = f(p) \cdot u$

Given multiplicative uncertainty of the type in equation 5.55, identically with additive uncertainty the cost function determines the outcome. The optimality condition – as in Leland – incorporates a second and third derivative (5.60b and 5.60c). Expected profits under uncertainty equal profits under certainty (equation 5.56), and do not influence the comparison of certainty and uncertainty (equation 5.57).

$$q = f(p) \cdot u, E(u) = 1. \quad (5.55)$$

$$Z = p \cdot f(p) \cdot u - c[f(p) \cdot u]. \quad (5.56)$$

$$Z_p = u \cdot f(p) + u \cdot p \cdot f'(p) - c'[f(p) \cdot u] f'(p) u. \quad (5.57)$$

$$Z_{pu} = f(p) + p \cdot f'(p) - c'[f(p) \cdot u] f'(p) - f'(p) \cdot u \cdot c''[f(p) \cdot u] f(p). \quad (5.58)$$

$$Z_{puu} = \underbrace{2f'(p)f(p)c''[f(p) \cdot u]}_{+} - \underbrace{f'(p) \cdot f^2(p) \cdot u \cdot c'''[f(p) \cdot u]}_{+} \quad (5.59)$$

$$c'' < 0 \text{ and } c''' \leq 0 \Rightarrow \hat{p} < p^*. \quad (5.60a)$$

$$c'' = 0 \quad \Rightarrow \hat{p} = p^*. \quad (5.60b)$$

$$c'' > 0 \text{ and } c''' \geq 0 \Rightarrow \hat{p} > p^*. \quad (5.60c)$$

5.3.3.4 *Multiplicative Uncertainty, Type: $p = g(q) \cdot u$*

The multiplicative uncertainty of the stipulated type implies a demand function in the form $q = g^{-1}(p/u)$ assuming that the inverse of $g(\cdot)$ exists. Both marginal revenues and marginal costs will now change under uncertainty. If marginal revenue is concave in u and marginal costs convex in u , both defined as derivatives with respect to p , optimal price will be reduced; in the opposite case it will increase. If both are concave or both convex, then the result remains indefinite. Conditions for concavity or convexity, respectively, remain the same as in the above-named cases.

$$p = g(q) \cdot u \Rightarrow q = g^{-1}\left(\frac{p}{u}\right). \quad (5.61)$$

$$E\pi = \int \varrho(u) \left\{ \left[p \cdot g^{-1}\left(\frac{p}{u}\right) \right] - c \left[g^{-1}\left(\frac{p}{u}\right) \right] \right\} du. \quad (5.62)$$

$$Z_{YXX} = \pi_{puu} = MR_{uu} - MC_{uu}. \quad (5.63a)$$

$$Z_{YXX} < 0 \text{ if MR concave in } u \text{ and MC convex in } u. \quad (5.63b)$$

$$Z_{YXX} > 0 \text{ if MR convex in } u \text{ and MC concave in } u. \quad (5.63c)$$

5.3.4 **Evaluation of the Outcomes of the Equilibrium Monopoly Models**

All presented models have in common that an *ex post* variable brings about a market equilibrium: all produced goods are sold and all demanded goods are supplied, there remain no involuntary stocks and no demand unsatisfied. Yet the results are manifold, less ambiguous than might be expected from the literature, and dominated by Leland's exposition. The

importance of the 'type of uncertainty' for the outcome necessitates a discussion on what is implied by the various assumptions and whether there are indications for a greater plausibility of certain assumptions.

An additive uncertainty means (in each of the two identical forms $p=f(q)+u$ and $q=g(p)+u$) an equal absolute amount of uncertainty under choice of high or low values of the action variables. The quantity-setting monopolist is unable to change absolute dispersion of the prices he has eventually to face, but relative dispersion may be reduced by selecting a smaller quantity hence equal absolute dispersion a higher expected price. Nor can the price-setting monopolist change the dispersion of quantities he may have to produce.

In multiplicative uncertainty there is a type of uncertainty for which the relative dispersion of the *ex post* control is independent of the choice of the action variable, and a type of uncertainty where relative dispersion declines with each smaller value of the action variable. The type of uncertainty that has this effect differs according to mode. For the quantity-setting monopolist the function $p=g(q)\cdot u$ results in a relative degree of uncertainty that cannot be influenced by the action: the relative scatter of contingent prices cannot be reduced even by a 'low' action. For the price-setting monopolist, the function $q=f(p)\cdot u$ will result in a relatively smaller dispersion of quantities to be produced, irrespective of chosen price. These two combinations of one type of uncertainty and one mode of behaviour were considered 'natural' by Leland and the authors who followed in his footsteps.

Conversely, there are two combinations of type of uncertainty and mode of behaviour which – for extensive ranges of demand functions (e.g. the linear and the quadratic – necessitate the choice of a low value of action variables, since this can reduce (absolute and relative) uncertainty. Given uncertainty in the form $q=f(p)\cdot u$, then the quantity-setting monopolist, by means of lower output, will reduce the dispersion of eventual prices. The price-fixing monopolist reduces the dispersion of outputs he will eventually have to produce by choosing a lower price. Due to technological fact (demand curve and cost curves, respectively), decisions will now be different from what they would be under certainty.

One cannot say in advance which type of uncertainty will be a more realistic description of real-world behaviour (there is no 'strong intuitive appeal' for either, therefore also not for the PIU). But at any rate the type of uncertainty (e.g. whether $p=g(q)\cdot u$ or $q=f(p)\cdot u$) does *not* depend on whether the monopolist is a price-setter or a quantity-setter (conversely, a dependence may exist where all modes are technically possible and are used according to profit maximization).

Thus, for example, given a price ceiling (be it fixed by regulation, approved by management/labour agreement or simply dictated by public opinion), we may expect the model $q=f(p)\cdot u$ to be realistic, and also that higher prices will show less dispersion.

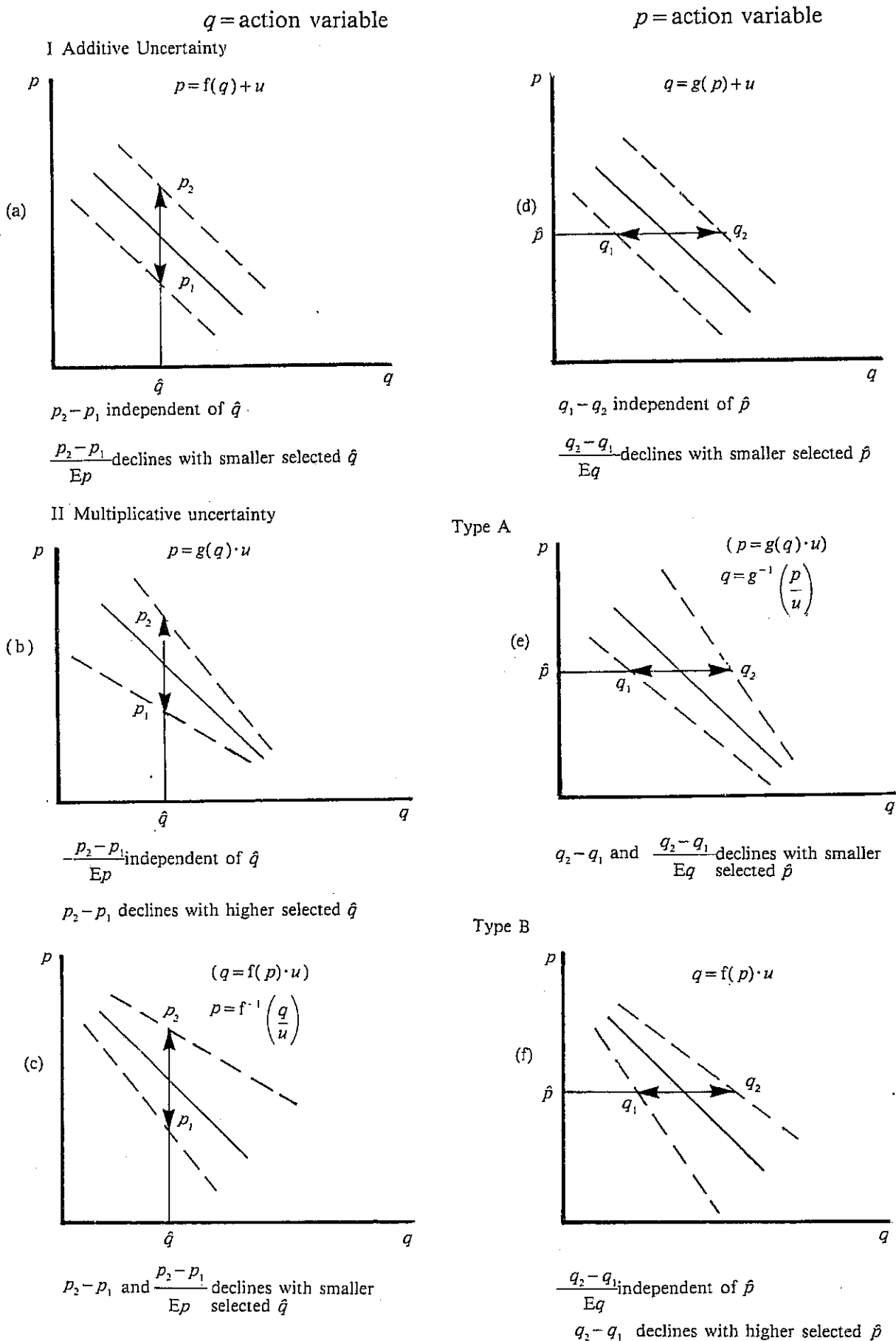


Figure 5.1 The six equilibrium models (two modes, and 3 types of uncertainty) of monopoly under uncertainty

Of the six models, the price-fixing monopolist will in principle be more susceptible to behave differently under uncertainty as compared to certainty (in all cases cost uncertainty is relevant, because of the unknown quantity to be produced), but there is little hope that universally valid empirical information can be gathered concerning the third derivative of the cost curve. The influence of the demand side is likely to be the more easily assessed (although it makes its appearance in only one model each under price and quantity uncertainty), since the condition of linear quadratic or constantly elastic demand functions appear relatively general. At least it may be assumed that the relevant subjective demand curve should hardly be more complex than linear or quadratic.

5.3.5 Monopoly, p - q -mode – Disequilibrium Model

A third 'behaviour' (mode) open to the monopolist consists in setting both price and quantity before the location of the actual demand curve becomes known. Mills (1962) has described this behaviour at an early stage, therefore it is also known as the Mills strategy. It is the behaviour providing least flexibility, since it is not possible – once actual demand becomes known – to react, either by price or by production adjustment; consequently market clearance cannot take place (unsold production or unsatisfied demand will result). This strategy is adopted when other strategies are not possible, but under certain circumstances (see Lim 1980) it may constitute a behaviour that may, at least as against the p -strategy, lead even to a higher profit than expected. The outcomes (we follow Karlin & Carr 1962) depend, at least in part, on the type uncertainty; equation 5.64 demonstrates a multiplicative form of uncertainty,⁷ equation 5.69 shows an additive form. The second component relevant is a factor comparable to the marginal costs of uncertainty as revealed in proposition 3.

$$\begin{aligned}
 x &= g(p) \cdot u \\
 E(u) &= 1 \\
 \phi(x) &= F\left(\frac{x}{g(p)}\right) \quad x \geq 0 \\
 f(x) &= \frac{1}{g(p)} \cdot f\left(\frac{x}{g(p)}\right). \quad (5.64)
 \end{aligned}$$

where ϕ and F are distribution functions.

$$x = g(p) + u, \quad E(u) = 0, \quad x \geq 0. \quad (5.65)$$

Borrowing from Karlin and Carr's derivation (with modification and adopting the present study's notation) the optimal price (\hat{p}) for the

multiplicative case will result from the solution of equation 5.66; optimal quantity is arrived at by solution of equation 5.67 (cf. Karlin & Carr 1962, p. 165).

$$(p - c) \cdot g'(p) + g(p) = - \frac{c \cdot g'(p) \cdot N(p)}{D(p)} \quad (5.66)$$

where $(p - c)g'(p) + g(p) = 0$ is that equation from which price p^* under certainty will result.

$$D(p) = \int_0^{F^{-1}(v)} [1 - F(\xi)] d\xi, \xi = \frac{x}{g(p)}$$

$$N(p) = D(p) - F^{-1}(v)$$

$$v = \frac{p - c}{p}$$

$$\hat{q} = g(\hat{p})F^{-1}\left(\frac{\hat{p} - c}{\hat{p}}\right). \quad (5.67)$$

The value p that satisfies equation 5.66 is always higher than the value under certainty (that value that results if the left side of the equation is zero), since the term on the right side is smaller than zero. Proof thereof (cf. Karlin & Carr): the denominator is positive, c is positive, $g'(p)$ – due to the negative slope of the demand curve – is negative, $N(p)$ is negative for all $p > c$.

$$g(\hat{p})F^{-1}\left(\frac{\hat{p} - c}{\hat{p}}\right) - g(p^*) \leq 0 \Rightarrow \hat{q} \leq q^*. \quad (5.68)$$

Optimal output q may be bigger or smaller than chosen output under certainty. Under very plausible assumptions, however, it is less than under certainty.

The reduction of optimal quantity results from two factors. Firstly, a lower optimal quantity follows from the higher price p (according to the negative slope of the demand curve under certainty), even if $F^{-1}(\cdot)$ equals 1. Second, the value of $F^{-1}(\cdot)$ is smaller than 1 if p is less than twice c . This latter is the slightly modified condition for the ratio of profit to costs, which was discussed in the competition model under demand uncertainty, and will be discussed in the newsboy model (cf. section 6.2). The modification adopted lies in that the higher optimal price \hat{p} – established by optimization (and not the fixed price) – is put in relation to costs.

For additive uncertainty, optimal price and optimal quantity are derived from equations 5.69 and 5.70, respectively.⁸

$$\hat{p} \text{ satisfies } (p - c)g'(p) + g(p) = \int_{F^{-1}(w)}^{\infty} [1 - F(u)] du. \quad (5.69)$$

$$\hat{q} \text{ satisfies } \hat{q} = F^{-1}\left(\frac{\hat{p} - c}{\hat{p}}\right) + g(\hat{p}). \quad (5.70)$$

The left side in 5.69 would have to equal zero under certainty, the right side is positive, therefore optimal price (\hat{p}) in the additive model must be lower than under certainty. Optimal quantity \hat{q} is undetermined. Two forces act in opposition to each other. Under very plausible assumptions the first term in 5.70 is negative, (since we maintain that profit, $p - c$, lies below c^9); on the other hand, price elasticity of demand will now militate in favour of higher output since price \hat{p} is now lower than under certainty (p^*). The condition for the increase – or reduction – of output under uncertainty is given in equation 5.71:

$$g(\hat{p}) - g(p^*) + F^{-1}\left(\frac{\hat{p} - c}{\hat{p}}\right) \cong 0 = q \cong q^*. \quad (5.71)$$

The present model imputes constant unit costs.

Nevin (1966) shows by numeric simulations that with rising unit costs the price may, also in the multiplicative model, lie below that under certainty.

The general tendency of multiplicative uncertainty rather leading to a price in uncertainty that lies above the price under certainty (thus strengthening a downward bias of output), and conversely, of additive uncertainty tending to lower the price (and therewith to a stricter condition for downwards distortion of output) may be explained (see Zabel 1972) by the fact that, given multiplicative uncertainty, a higher price will also reduce uncertainty (assume, for example, an excessively high price; then demand – and with it also uncertainty about demand – will, in the multiplicative case, drop towards zero).¹⁰

5.4 UNCERTAINTY ABOUT PRODUCTIVITY (PRODUCTION UNCERTAINTY)

Uncertainty may exist not only about prices or demand but also about the productivity of factor inputs. Breakdown of machinery may reduce productivity of capital, absenteeism or strikes may reduce productivity of labour. We have to distinguish in this chapter between labour, capital and capacity installed (for which we will use the letters L , K , and C) and labour, capital and capacity rendered (L_1 , K_1 and q); we will use the term capacity for the maximum production feasible with the factors chosen if we do not relate output to the individual factors but to the combined inputs.

It is no surprise that the same forces which lead to divergent results for price and demand uncertainty are working also in a world of production

uncertainty. Market structure, availability of an *ex post* control and the chosen type of uncertainty decides about the result.

Taking a model with given prices and given demand (\bar{x}) which may be interpreted as a model with production on order), expected profits depend on the smaller of order and capacity rendered (equation 5.72). Capacity rendered (q) depends on capacity installed and a random variable u for the uncertainty about the productivity of the combined factor. Optimal capacity installed is smaller than under uncertainty (equation 5.73), due to the extra cost, for example of potential breakdowns of machinery.

This result survives in an economically more important model where demand is the second random variable, whose density function $f(x)$ is known (equation 5.74). The lower capacity planned in this case ($\hat{C} < C^*$) is due to the combined effect of potentially unsold production (in case of low demand) and the irregularities in the productivity (equation 5.74). The characteristic features of the model are the fix-price assumption and that inputs can be described by 'capacity'. There is no *ex post* control which closes disequilibria after the actual values of the random variables become known.

Production uncertainty model with given demand \bar{x} :

$$E\pi = \int_{-\infty}^{\bar{x}-g(q,C)} p \cdot q(u,C) f(u) du - \int_{\bar{x}-g(q,C)}^{\infty} p \cdot x f(u) du - c(C)$$

$$q = q(u,C) \Rightarrow u = g(q,C) \quad E[q(u,C)] = q(C). \quad (5.72)$$

$$p \cdot q'(u,C) \cdot \underbrace{F(\bar{x} - g(q,C))}_{< 1} = c'(C)$$

$$c'(\hat{C}) < p \cdot q'(u,C) \Rightarrow \hat{C} < q^*, \quad (5.73)$$

e.g. if q is linear: $c'(\hat{C}) < p$.

Production uncertainty with unknown demand ($f(x)$):

$$E\pi = \int_0^{\bar{x}-g(q,c)} \int_{-\infty}^{\infty} p \cdot q(u,C) f(u) f(x) dx - \int_{\bar{x}-g(q,c)-\infty}^{\infty} \int_{-\infty}^{\infty} p x f(u) f(x) dx - c(C). \quad (5.74)$$

$$\underbrace{\int F(x - g(q,C)) f(x) dx}_{A < 1} \cdot q'(u,C) = c'(C) \Rightarrow \hat{C} < q^*,$$

$$\text{e.g. if } q \text{ is linear } \Rightarrow p = c'(\hat{C}). \quad (5.75)$$

Assuming the more standard competitive assumption that all production can be sold (though we first hold to the assumption of price stickiness), the impact of production uncertainty hinges on the type of uncertainty. For multiplicative of the type $q(C \cdot u)$ and additive uncertainty there is no impact; for a more general form $q(C, u)$ it depends on the concavity, linearity or convexity of the marginal revenue.

$$E\pi = p \cdot E[q(C, u)] - c(C) \quad (5.76)$$

if $q(C, u) = q(C + u)$ or $q(C, u) = q(C \cdot u) \Rightarrow \hat{C} = q^* = C^*$ (since $Z_{Cuu} = 0$)

if $MR_{uu} \leq 0 \Rightarrow q^* \leq C^*$.

Other models relax the assumption that production uncertainty effects both inputs in the same way (as we did up to now modelling 'capacity'), now the substitutability of the production function becomes relevant. Ratti & Ullah (1976) and Roodman (1972) construct models where strong qualitative results can be derived; the assumptions necessary to do this have been criticized as too narrow by Pope & Just (1978).¹¹ Turnovsky (1973) and Feldstein (1976) have presented more general models, but were not able to derive strong qualitative results.

Roodman (1972) assumes a fixed coefficient production function. One of the two factors employed is reliable, so that service chosen is identical with service rendered; the second is unreliable in the sense that $F_1 = u \cdot F$, i.e. a service chosen is multiplied with a random variable with unit mean. It can be shown that expected output will be smaller than under uncertainty though the quantity of the unreliable factor chosen will be larger. The rationale for decreasing output under uncertainty is the extra costs of the reserve capacity. Roodman's model fits into our disequilibrium models, since there is no *ex post* control which can be adjusted after the veil of uncertainty has lifted, the extra cost of uncertainty guarantees an unambiguous result.

Ratti & Ullah (1976) investigates uncertainty about the productivity of labour and capital in a competitive model. The firm can choose the inputs of capital and labour (K, L), the actual services rendered by the inputs however, are uncertain ($K_1 = u \cdot K, L_1 = v \cdot L$) and therefore the output is also a random variable. u and v are assumed to have a unit mean, and no relation between them is defined. Ratti and Ullah show that, under the condition (i), that the elasticities of the marginal product curves² are as defined in equation 5.77a non-increasing functions of factor services (see equation 5.77b), and second condition (ii), that the factors complement one another less and less as more of each factor is employed (see equation 5.77c), the input demands of firms operating in an uncertain environment are less than under certainty and so is output:

$$\eta = \frac{K_1 f_{K_1}}{f K_1}, \quad \epsilon = \frac{L_1 f_{L_1}}{f L_1} \quad (5.77a)$$

$$\frac{\delta \eta}{\delta K_1} \leq 0, \quad \frac{\delta \epsilon}{\delta L_1} \leq 0. \quad (5.77b)$$

$$\frac{\delta K_1 L_1}{\delta K_1} < 0, \quad \frac{\delta f_{K_1 L_1}}{\delta L_1} < 0. \quad (5.77c)$$

This rather strong result is derived from the fact that formally the uncertainty variables are placed 'inside' the production function, therefore the functional form (concavity) of the production function helps to produce results diverging from the certainty case even in cases of risk neutrality. Higher and lower actual services from the chosen input factors are distortions from the optimal input combinations under certainty and these extra costs are 'severe' in the sense that there is no variable which can adjust after the factor service rendered is known.

5.5 CONCLUSIONS

The models presented in this chapter have put the current paradigm of uncertainty theory that 'risk aversion neutrality loving decreases leaves constant increases' optimal decision into perspective. Risk proneness decides about the result if profits are linear (as shown in proposition 1) and this is the case in the competition model with price uncertainty, or in the monopoly equilibrium model with price-taking behaviour and a special type of uncertainty (additive or multiplicative type A).

The results of the model in general however are much richer, since all kinds of technological concavity (convexity) tend to divert optimal results under uncertainty away from the certainty results. For example, in the case of other monopoly models, elements of the cost function or the demand function decide on the outcome. These effects could be exaggerated or mitigated by non-linear utility functions, but the models following proposition 2 show an economic importance of uncertainty independent of the risk attitude.

In models in which prices do not adjust rapidly enough to clear the market, the possibility of shortages and overcapacity becomes the important feature. Risk aversion is not necessary at all to bias optimal output downwards under competition with demand uncertainty, under monopoly with disequilibrium or in models with uncertainty about the productivity. Models in line with proposition 3 therefore show the most clear and unambiguous impact of uncertainty (without any reference to risk aversion). Critics of this type of model will maintain that price stickiness may be suboptimal, one of the principal questions for empirical research is therefore whether these models represent real-world behaviour and why a certain degree of price stickiness may have very economic reasons.

The models presented up to now concentrated on the one-period optimization, on one-decision variable only and on the strict divide between one variable which had to be decided upon *ex ante* without any feasibility to change the decision. The following chapters will deal with these shortcomings one by one.

NOTES

- 1 More accurately: uncertainty about output price.
- 2 Hey (1979) rejects quantity fixing under demand uncertainty for competition since profit would be higher in this case if the firms were to be set the price.
- 3 The term 'linear technology' is used according to the definition presented in section 4 ($Z_{YXX}=0$), not in the narrow sense of a linear cost and production function.
- 4 In one of the first articles of firm's behaviour under uncertainty, Leland (1972) characterized its form thus: that uncertainty ought to increase with rising expected sales (principle of increasing uncertainty - PIU).
- 5 Cf. Leland (1972); though he points out that the outcomes should tendentially remain intact if this is taken into consideration. This study, in conformity with the prevailing literature, assumes that demand will be satisfied, given available output.
- 6 The potential violation of PIU shall be demonstrated for the quantity-setter and

for the alternative functions $p(q,u) = (q) \cdot u$ and $p(q,u) = h\left(\frac{q}{u}\right)$. As shown by

Leland, PIU is equivalent to stating that (following the terminology of this study) R_{qu} has the same algebraic sign as $E(R_q)$. In the first case, R_{qu} and ER_q are identical and therefore also have the same algebraic sign.

In the second case $R_{qu} = h'\left(\frac{q}{u}\right) \frac{-q^2}{u^2} + q \cdot \frac{1}{u} \cdot h''\left(\frac{q}{u}\right) \frac{-q}{u^2} + q \cdot h'\left(\frac{q}{u}\right) \frac{-1}{u^2}$,

where the first and third terms are positive, and also the second under concavity of the demand function (therefore the whole term also under linear demand function).

$ER_q = E \left[h\left(\frac{q}{u}\right) + q \frac{1}{u} h'\left(\frac{q}{u}\right) \right]$, the first term being positive and the second

negative. Under linear demand curve of the type $p(q \cdot u) = \frac{q - a \cdot u}{bu}$, $b < 0$ R_{qu}

is in any case positive $\left(R_{qu} = \frac{2a}{b} \cdot \frac{-1}{u^2} \right)$, $ER_q \frac{2q}{b} E \frac{1}{u} \frac{q}{b}$ only if

$\left| 2 \frac{q}{b} \left(\frac{1}{u} \right) \right| < \left| \frac{-a}{b} \right|$, which is not generally the case.

- 7 The type of multiplicative uncertainty is not of the same importance in the p - q -mode as in equilibrium models, because one differentiates simultaneously with respect to p and q .
- 8 It is always assumed that price is 'sufficiently' high in the sense of $p > c$, and $\hat{p} > c$ respectively.
- 9 In this case the lower price (lower as compared with certainty), enters into the calculation.
- 10 Zabel shows also that - qualitatively - the statements and effects of uncertainty remain similar in the one-period model and the dynamic model, but that stricter conditions are needed in the multiplicative case, in order to guarantee that the solution be unambiguous. In another article (Zabel 1970, p. 215), Zabel deals with the difference between constant and rising marginal costs. In optimization conditions for the dynamic case Karlin & Carr exclude the feasibility of backlogging and obtain a value as in equation (6.27) in section 6.7. In our opinion, use of the right side of (6.14) would be more realistic.

- 11 Pope and Just criticize the assumption that capital rendered is a multiplicative function of capital installed. This implies that the variance of capital rendered increases also with higher capacity installed, which contradicts some notion of greater flexibility. I do not find this criticism very convincing, since flexibility does not refer to the one input alone but to the substitutability of two inputs, and this effect is fairly dealt with in the model.

14 The Main Findings of the Book

1 Uncertainty theory offers the possibility, that 'cautious' decisions can be considered as rational under a number of circumstances and that real economies are probably working under conditions where these circumstances will hold. We have formally derived under which conditions the optimal value of the decision variable chosen under uncertainty has to be smaller than under certainty – or if the latter is not available under which condition the optimal value will lie below expected demand. If there exists some *ex post* flexibility we can derive from these also the conclusion that optimal anticipations (plans, expectations) can lie below realizations even in the long run (on average). Downward-biased reported expectations can be – and under realistic empirical circumstance are – economically rational.

2 Following the mainstream theory we model behaviour under 'risk' and not under 'uncertainty proper', under which (according to the old dichotomization of Knight) no probabilities can be assessed for the different states of nature. We furthermore assume that the axioms of the Neumann–Morgenstern Utility Theory apply. We made these two assumptions, not because we believe that they are more realistic than alternatives either proposed by Keynesian macroeconomists or by critics of expected utility maximization, but because we believe that these methods constitute the stricter test for the casual notion of the businessman 'that we should behave cautiously under uncertainty'. Nevertheless we try to bridge the gap between standard mathematical theory and (Post) Keynesian economists since we allow maintained arguments of Post-Keynesians (for example, that prices tend to be rigid under uncertainty or that flexibility and liquidity gain importance), to influence the model selection. And if these stylized facts are accepted we can show that Keynesian results can be derived from models even with expected utility maximization. The conciliation will remain incomplete, however, since neoclassical economists will question the rationality of price stickiness and at least defy its importance for normative questions, while Keynesians will think that too much of the assertion of 'totally different' behaviour under uncertainty is lost if they accept the Procrustean bed of expected utility maximization.

3 Within the risk models we arrive at the conclusion that the role uncertainty exerts seems to differ qualitatively depending on some *a priori* assumptions. In models where we did not allow an *ex post* control to bridge differences between supply and demand, in which prices do not adjust, and in which unused capacity cannot be disposed of, uncertainty exerts a more substantive effect on the decision, and the direction of the influence could be easily assessed. The same holds true for large one-shot decisions in contrast to repeated small decisions. In models with *ex post* controls the impact of uncertainty depends on facts difficult to evaluate empirically (like third cross-derivatives). We used this finding tentatively to propose a new *dichotomization* of uncertainty models into those with 'severe' uncertainty and those with 'petty' uncertainty. These dichotomizations show some resemblance to the old 'uncertainty-risk' dichotomy and may be used to describe the controversy between Keynesians and neoclassical economists. The decisive criterion, however, is not whether a probability function can be assessed, but whether there are *ex post* controls whose *ex post* adjustment can reduce the imbalancing effects of decision errors.

4 Four sources for the rationality of 'cautious' decisions have been identified:

- risk aversion implies a lower optimal value under uncertainty. The two preconditions – a linear technology and a positive relation between the optimal decision value under certainty and the random variable (dY^*/dX) – are sometimes forgotten in the literature, but overall this effect is well documented. We repeat this as a sort of reminder, we did not follow this path in detail, since it is the very preoccupation of literature with risk attitude which prevented the literature from investigating more objective (technical) reasons for the influence of uncertainty;
- technological concavity proper: given linear utility function, whether Z_Y (the certainty optimum) is concave, linear or convex in the random variable, decides in which direction optimal decision under uncertainty differs from certainty. Technological concavity (Z_{YXX} smaller than zero) may stem from the cost function, production functions characteristics, demand curve, etc. Though this rather general condition can be derived quite easily, it is not elaborated in literature;
- marginal cost of uncertainty: if there is no *ex post* control which closes a difference between supply and demand, the expected costs of these imbalances have to be incorporated. Here a cost component enters the calculation which had not been present under certainty. The results deriving from such models are much more robust (they do not need an evaluation of a third cross-derivative), but, on the other hand, the question of the rationale for this lack of *ex post* control arises. Prices have to be assumed not only to be sticky but also to be identical under certainty and uncertainty; and

- asymmetrical *ex post* flexibility: if it is feasible to revise a decision during the planning period and if this revision is easier (less costly) in the upward direction, this leads to a cautious first plan for the decision variable. Any of the three reasons mentioned before plus an easier upward revision of preliminary decisions may result in 'cautious' first plans, repeatedly lower than the *ex post* realizations.

5 Perfect neoclassical models with rapid price adjustments, no imbalances, perfect future markets, and total reversibilities tend to produce results with no (or arbitrary) effects of uncertainty. For example, risk aversion may create a downward bias in the competition model under price uncertainty. This can be partly reduced by the introduction of future markets. Models with rigidities (competition with demand uncertainty, *p-q* monopoly, irreversibility of investment, *a priori* fixed capacity) result more often in optimal decisions *lower* under uncertainty.

6 The strongest theoretical pressure for *higher* optimal values under uncertainty operates for input factors. Here technological convexity results from possible higher expected profits in the case of possible positive realizations of uncertain demand. The optimality of 'large' investment programmes in case of uncertainty is grossly at variance with the well-documented tendency that investment plans are on average revised upwards and not downwards.

7 The operationalization of the notion of rational expectations as the conditional expected value implicitly assumes a linear objective function. If the only reason for non-linearity were the risk attitude, this would not seem to be overly restrictive. Since we purport that the main elements of non-linearity may be technological *in the wider sense*, we should try to derive more general concepts of rationality. We derive the concept of economically rational expectations, which incorporate the consequences of errors into the formation of expectations. *Economically rational expectations* are decisions (actions) in the terminology of decision theory. We think that sales' anticipations formed by firms, and forecasts made by macroeconomists are approximations of this concept of economically rational expectations since they are not formed without explicitly or implicitly considering errors. Reported expectation in surveys may now deviate from realizations not due to psychological factors, planning errors, or sampling biases, but due to very economic reasons (one of the four sources as mentioned in chapter 4). Annex 3 demonstrates that non-linearities destroy the main outcome, derived by the application of mainstream rational expectation to economic policy, namely the impotence result.

8 In the standard model of competition under price uncertainty only risk aversion (loving) can distort the decision from its equivalent under

certainty. With risk neutrality, uncertainty is unimportant. In the outsider model of competition under demand uncertainty, uncertainty unambiguously biases the decision downward due to the additional cost component of potentially unsold production. This model may comprehend elements of oligopolistic behaviour or may capture behaviour in regulated markets. The price stickiness however – albeit necessary only in the very short run – may be considered to be at variance with the spirit of competition.

9 Following Leland, the literature informs us that in the monopoly model with price as an *ex post* control, optimal quantity is independent of uncertainty. This is not true for a type of multiplicative uncertainty (type B in section 5.3) where price uncertainty decreases with higher price. Here optimal production is lower under uncertainty under quite general conditions (linear or quadratic demand).

In the monopoly model without *ex post* control (*p-q*-mode), multiplicative uncertainty tends to induce a higher price and a lower optimal output; in models with additive uncertainty empirical parameters decide about the outcome. We argued in the empirical part that these parameters will lie in a range yielding a lower output under uncertainty in this model too.

10 Inventory theory yields a fruitful complement to the one-period models since it takes the value of the imbalances at the end of the (first) period into account. Usually inventory models start with fixed prices, in this case the comparison may be done only with expected demand. The optimal stock on hand (inventory after production and before demand) will lie below or above expected demand depending on the relative importance of costs to profits, of goodwill to holding cost, and of backlogging feasibility *v.* durability.

The results of this part of the book contradict prevailing literature in the following ways:

- we show that the asymmetry of profits to cost (the first are much lower per unit than the latter) creates a strong tendency for small stocks on hand (and also for small production). In the newsboy model this asymmetry decides alone; in a dynamic model it is still a decisive force (together with the other parameters mentioned above);
- we insist that at least in all theoretical models the feasibility of backlogs should be incorporated, its *a priori* neglect biases the results in a very serious way; and
- we show how the implicit prejudice of inventory theory – that uncertainty increases inventories – is far from evident. Most probably the contrary is true. We correct formulae presented in articles and textbooks on the forgotten revenues from backlogged sales.

11 In general – this duplicates a lot of parallel findings in the literature – the results of static and dynamic models do not differ too much. Dynamic

models bring some additional insights, the results tending to lie between that of the certainty model and the static model under uncertainty (the costs of uncertainty can be partly recovered by repetition). The intuitive notion, that production in a dynamic model must be higher (e.g. Mills 1962) since inventories have a value, is misleading, since on the other hand backlogged demand has a value too.

12 One of the critical facts theory points out and which has to be evaluated empirically is the degree of price stickiness in modern industrial production. Uncertainty will yield different decisions as compared with certainty if prices are sticky, albeit this has to be only a short-term stickiness and partial stickiness. Only if prices are flexible in the sense of immediately offsetting demand shocks do no disequilibrium costs have to be incorporated. We have demonstrated empirically by econometric and by survey methods that:

- industrial prices tend to reflect cost conditions and may be some price signals stemming from international markets, but do not react rapidly to demand shocks;
- prices seem to be less variable than quantities, especially the short-run fluctuations;
- price expectations are not less inaccurate as compared with production expectations (as they should be if the prices were an *ex post* control);
- price rigidities are at least as dominant in that sector of manufacturing for which a large number of enterprises, their small average size, etc., would suggest more competitive behaviour than for the rest; and
- asked about their response to a demand shock only 20 per cent of the firms in a survey cited price change as primary response.

13 On the other hand, quantities seem to be more flexible than assumed in most models, be it that production is an *ex post* control or be it that a preliminary production can be decided upon. Then, after demand is revealed, the decision variable can be partially adjusted (*ex post* flexibility); as indicators for a partial adjustment of quantities with a remaining part of disequilibria we found:

- the capacity utilization of industry as well as the inventory sales ratio fluctuates to a considerable extent, and surveys tell us that firms do consider these fluctuations as involuntary;
- output volume follows demand shocks closer than prices; and
- asked about reaction to demand shocks, 56 per cent of the firms labelled quantity changes a primary response, and 48 per cent reported changes in inventories (part of the disequilibrium is maintained).

14 In general, *ex post* flexibility seems to be greater than suggested in standard models. The distinction between *ex ante* variables, which have to be decided before the veil of uncertainty is lifted and *ex post* variables, which have to be decided (or which adjust) thereafter is not watertight.

However, *ex post* flexibility of quantities seems easier than that of prices (in contradiction to theoretical assumptions).

15 *Ex post* flexibility seems easier upwards than downwards. This seems especially true for input decisions, where this tendency for reported investment anticipations can easily be demonstrated on the micro and macro level. This asymmetry overcomes the theoretical tendency of uncertainty to increase optimal investment. From the theoretical viewpoint this source of bias is not very attractive. To model an asymmetry sounds *ad hoc*, if it is empirically true, the theoretical implications are trivial.

16 The mainstream model of competition with price uncertainty is exposed as an outsider in a real economy. Only 7 per cent of the firms consider it as relevant to their situation; a monopoly model with price as *ex post* control is chosen by 16 per cent. The remaining majority reported that prices are not an *ex post* control; quantity-setting and a partial *ex post* adjustment of the quantity set are considered as the most realistic models.

17 If we should draw a picture of a standard 'representative firm' in modern industry we would do so in the following way: industrial firms have to decide on a preliminary production, they set up a cost price (or accept a market price). In case of demand shocks, output is partly adjusted, partly backlogs and/or inventories are changed, prices change slowly and in response to large stocks. Adjusting quantities upwards is easier/less costly than adjusting them downwards. At least sources three and four (marginal cost of uncertainty and asymmetric *ex post* flexibility) usually tend to bias (preliminary) decision under uncertainty downwards, technological concavity may add some other source of asymmetry.

18 Risk attitude is the most popular channel in the literature but most difficult to assess. If the results of entrepreneurs in a survey may be considered reliable, it looks as if entrepreneurs behaved as if they were risk-neutral for small, repeated decisions and risk-averse for large, one-shot decisions.

19 Optimal starting stocks depend positively on the profits, goodwill costs and the durability of stocks, and negatively on production costs, holding costs, backlogging facility and the discounting factor. In the empirical part we showed that:

- profits are much lower than production costs;
- firms allow their inventories more frequently to go down than to pile up; and
- backlogged demand – as measured by order stock – is much higher than finished-good inventories.

These empirical facts would indicate that optimal stocks on hand should decrease with uncertainty and lie on average below expected demand.

20 Inventory fluctuations are shown to conform with the main features of the models presented. Profits are shown to be an important determinant, price speculation not (hinting that inventories' fluctuation does not result from optimizing activity, rather they are a disequilibrium phenomenon). Inventories declined, in a period of presumed greater uncertainty and backlogs increased.

21 We arrive at the conclusion that the dominant dichotomization of uncertainty literature into 'risk' and 'uncertainty proper' (according to the criterion whether probability functions about the uncertainty variable can be formed or not) is not very fruitful, since in the latter case only very crude rules of behaviour can be derived in a coherent and consistent way. The Keynesian view, that economic decisions are done in an environment much more complex than in an optimization problem where one certain variable is substituted by one for which a probability function is known, is nevertheless a useful warning. That no probability function can be assessed (or used implicitly) is an extreme alternative however, and precludes the economic analysis of a large area of economic problems.

We believe that it is important how the decision model is constructed, whether the importance of uncertainty will be considerable or minor, not whether we assume that probabilities can be assessed. If we construct models in which disequilibria exist and are not instantaneously closed by some *ex post* control, if we model the decision process as choosing between alternative techniques and degrees of flexibility, then we can use Neumann-Morgenstern's expected utility theory in general and probability functions and nevertheless describe a situation in which people behave 'qualitatively differently' under certainty and uncertainty.

We tentatively propose that the real divide between uncertainty that matters and uncertainty with less consequences is whether there are chances to correct a decision (or at least to make errors in some way unimportant). This correction can either be a two-stage optimization process (short-run optimization for a given long-term optimization, e.g. for labour and capital), or it can be that the market price adjusts automatically yielding equilibrium for any quantity decision or that goods are durable so that unsold production can be used in the next period. We propose to label situations in which such adjustments are feasible as '*petty*' uncertainty, since the importance of uncertainty is mitigated to a large extent by these strategies. Models in which there are less strategies for *ex post* adjustments are labelled as '*severe*' uncertainty, since they usually result in disequilibria with important medium- or long-run consequences.

22 We want to stress some limitations of our book:

- we assumed expected utility maximization (EUM). Assuming other models may lead to divergent results, we still think that EUM is a strong test for our findings;
- in the theoretical part a whole variety of models had to be presented, many could have been added;
- we were concerned only with the optimization for the individual firm and did not derive consequences for industry or total economy;
- especially modelling *ex post* flexibility showed a large discrepancy between models available and their presumed empirical importance. Work has to be done, especially in simultaneously deciding the optimal degree of flexibility and the optimal decision;
- out of inventory models we had to make some rather restrictive assumptions like infinite horizon, i.i.d. demand, fixed prices, (sometimes) linear cost, and the absence of delivery lags;
- in the empirical part we had to concentrate on some crucial issues, among them price stickiness and inventory behaviour, empirical importance of disequilibria;
- as sources of information we had to rely on econometric analysis of aggregate data, on surveys and market results. Problems of aggregation of differences between individual optimization market results have to be assumed as not dramatically changing our result; and
- sometimes information was restricted to Austrian data; but wherever possible we used information from the USA, Japan and countries of the European Community.

23 If we want to sum up the results of this book in one paragraph – bearing in mind the limitations presented – we think that the theoretical feasibility of cautious production under uncertainty may for realistic conditions of industrial production come into effect due to price stickiness, disequilibria, backlogs and *ex post* flexibility. Production will most probably be *lower* under *uncertainty* than under certainty, this will be true *a fortiori* for optimal preliminary plans in a world of easier upward revision. *Economically* rational expectations may well persistently lie below actual outcomes.